Abstract: A new and improved HWTC apparatus for thermal conductivity measurements of common fluids and nanofluids has been recently developed, designed and fabricated. A platinum (Pt) wire of 50.8 \( \mu m \) diameter with a Teflon insulation coating of 25.4 \( \mu m \) thickness was used as the hot-wire heater and temperature sensor for the present application. The new apparatus employs innovative solutions for easy calibration of uniform Pt-wire tension and thus minimizing the strain influence on temperature measurement (i.e., minimizing the well-known and unwanted “strain-gage effect” on Pt-wire electrical resistivity); measurement of Pt-wire voltage drop independently from power wiring (four wires); and an effective off-centered mechanical design to minimize the test fluid sample size (about 35 mL), but at the same time providing additional space for wiring, including three inside thermocouples for fluid temperature uniformity verification. Data acquisition hardware and LabVIEW application software are optimized to minimize signal noise and enhance acquisition and processing of useful data.

The bias measurement error, based on calibration with distilled water and ethylene glycol, has been found to be within 1.5 %, and precision, i.e., repeatability error within 2.5 %. The total uncertainty in measured thermal conductivity (TC) has been estimated to be within 3.5 % at 95 % confidence probability. Two nanofluids of 1 % volumetric concentration of 35 nm size copper nanoparticles in ethylene glycol and in water, were prepared and their thermal conductivity measured using the apparatus. The mean enhancement in thermal conductivity of copper in ethylene glycol and copper in water nanofluids, were measured as 13 % and 16 %, respectively. The developed apparatus has proved to be efficient and with potential for further improvement.

Keywords: Bias error, data acquisition, measurement uncertainty, nanofluids, platinum hot-wire, precision error, thermal conductivity, transient method.

1. Introduction

The transient hot-wire technique is known to be a fast and accurate method for fluid thermal conductivity measurements. The principle of the hot-wire method is based on ideal, constant heat generation source, an infinitely long and thin continuous line, dissipating the heat into an infinite test medium. In practice, the ideal case is approximated with a finite long wire embedded in a finite medium. While the wire is electrically heated, the change in resistance of the wire, thus its temperature, is measured as function of time using a Wheatstone bridge circuitry and a computerized data acquisition system. Finally, the thermal conductivity (TC) value is determined from the heating power and the slope of temperature change in logarithmic time.

A new and improved, transient hot-wire thermal conductivity apparatus has been developed to measure the thermal conductivity of fluids, polymer solution, nanofluids and poly-nanofluids (a mixture of nano particles, polymers and conventional heat transfer fluids) [1,2]. The apparatus is a part of a research program at Northern Illinois University with an objective to resolve some critical issues in nanofluids research and to develop and optimize new hybrid, drag-reducing polymer-nanofluids with enhanced thermo-physical and tribological characteristics [2,3]. Thermal conductivity, a measure of material’s ability to conduct heat, is a very important property for thermal analysis and design.

Different methods have been developed to measure the thermal conductivity of regular fluids and
nanofluids [4 to 15], as described and discussed elsewhere [1].

The mathematical model for the hot-wire method is based on an ideal, infinitely long and thin continuous line source dissipating heat, of heat flux \( q \) per unit length, applied at time \( t = 0 \), in an infinite and incompressible medium. The general assumption is that heat transfer to the infinite medium, of thermal conductivity \( k_f \) and thermal diffusivity

\[
\alpha_f = k_f / \rho_f C_f ,
\]

is by conduction alone and thus increases the both temperatures in time, of the heat-source and test-medium. It is also assumed that the line heat-source has uniform instant temperature everywhere, but transient in time (virtually achieved with small diameter and long wire with large thermal conductivity and/or small heat capacity). The governing equation is derived from the Fourier’s equation for one-dimensional (1-D) transient heat conduction in cylindrical coordinates,

\[
\frac{1}{\alpha_f} \frac{\partial T}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) \tag{1}
\]

Where, \( T = T_0 + \Delta T \) is the temperature of the medium at any time \( t \) and arbitrary radial distance \( r \), \( T_0 \) is the initial temperature of the source and medium, and \( \Delta T \) is the temperature difference between the medium and initial temperature. The Eq. (1) is the subject of the following boundary conditions:

\[
\lim_{r \to 0} \left( r \frac{\partial T}{\partial r} \right) = - \frac{q}{2 \pi \alpha_f} \quad \text{at} \quad t = 0 \quad \text{and} \quad r = 0 , \tag{2}
\]

\[
\lim_{r \to \infty} [\Delta T(r, t)] = 0 \quad \text{at} \quad t \geq 0 \quad \text{and} \quad r = \infty . \tag{3}
\]

Where, \( \rho_f \) and \( C_f \) are density and specific heat capacity of the test medium, respectively. The infinite series solution of this problem is outlined by Carslaw and Jaeger [16]. After initial, short transient period (i.e., \( t >> r^2 / 4 \alpha_f \)), except for the first term containing time \( t \), the higher order terms could be neglected, resulting in a very good approximation as,

\[
\Delta T = T(r, t) - T_0 = \frac{q}{4 \pi \alpha_f} \left[ - \gamma + \ln \left( \frac{4 \alpha_f t}{r^2} \right) \right] \tag{4}
\]

where \( \gamma = 0.5772 \) is the Euler’s constant.

For constant fluid medium properties and a fixed and arbitrary radius \( r \), after differentiation of Eq. (4), the radius is eliminated from the equation, and the following relation is obtained,

\[
k_f = q \frac{1}{4 \pi} \frac{d(\Delta T)}{d \ln(t)} \tag{5}
\]

Therefore, if temperature of the medium is measured as function of time at any fixed radial position, including at the contact with the line source (i.e. the temperature of the ‘thin’ line source), the thermal conductivity of the test medium, \( k_f \), is proportional to the source heat flux and inversely proportional to the temperature (or temperature difference) gradient with regard to the natural logarithm of time, see Eq. (5).

The advantage of the hot-wire method is its simplicity and consequently low cost of construction. Furthermore, the wire itself acts as both the heating source and temperature sensor for measurement. Another advantage is that convection heat transfer effects can be minimized and identified when present as deviation of the linearity in the plot of \( \Delta T \) as a function of \( \ln(t) \).

2. Application of Hot-Wire Method for Nanofluids

A bare metal wire centered in a fluid medium is generally used for thermal conductivity measurement of fluids by the transient hot-wire method. Nanofluids containing metal particles are electrically conductive, so application of a bare wire could lead to ambiguous results in the measurements. Some of the problems identified by Nagasaka and Nagashima [17] in the application of the ordinary transient hot-wire method to electrically conducting liquids are: (a) possible current flow through the liquid, resulting in ambiguous measurement of heat generated in the wire; (b) polarization of the wire surface; (c) distortion of output voltage signal due to influence of the conducting liquid cell.

In order to overcome these errors, it is recommended that the bare metal wire should be coated using electrically insulating material. The effect on temperature distribution due to thin insulation coating has been analyzed by Nagasaka and Nagashima [17] and outlined by Yamase et al. [18].

The temperature rise \( \Delta T \) of the hot-wire is given as,

\[
\Delta T = \frac{q}{4 \pi k_f} \left[ \ln(t + A_o + \frac{1}{t} (B_o \ln t + C_o)) \right] \tag{6}
\]

The terms \( A_o \), \( B_o \) and \( C_o \) are defined as follows:
where, \( r_w \) is the radius of the wire; and \( r_o \) is the sum of the radius of the wire \( r_w \) and the insulation thickness \( \delta_i \). Subscripts \( w, I, \) and \( f \) represent wire, insulation coating, and liquid, respectively.

Comparison of Eq. (4) and (6) indicates that the term \((1/t)(B_o \ln t + C_o)\) is due to the presence of the insulation layer on the wire. If the term \((1/t)(B_o \ln t + C_o)\) is negligibly small compared to the \((\ln t + A_o)\) term, then the constant \( A_o \) shifts (i.e., offsets) the plot of \( \Delta T \) against \( \ln(t) \), without changing the slope. Therefore, the thermal conductivity, \( k_f \), is again accurately determined using Eq. (5).

Yu and Choi [19], have analyzed the wire temperature rise as a function of time, to determine the influence of insulation coating on the thermal conductivity measurement and have concluded that relative measurement error of thermal conductivity is negligible if the slope of \( \Delta T \) against \( \ln(t) \) is measured at later times after start of heating, and that no correction to insulation coating is necessary, even if the insulation coating thickness is comparable to the wire radius.

Electrical insulation coating to bare metal wire has been recommended for electrically conducting fluids. Nagasaka and Nagashima have coated the platinum wire (diameter 25 \( \mu \text{m} \)) with a Teflon insulation coating (thickness 1.5 \( \mu \text{m} \)) to measure thermal conductivity of various combinations of nano crystalline material and base fluids [13].

### 3. Hot-Wire Cell Design

The cell design was aimed to reduce the test sample volume of nanofluids, and at the same time, to provide a controlled tension in the hot-wire during heating and thermal dilatation. An innovative solution to easily calibrate the hot-wire tension and thus reduce strain influence on wire temperature measurement has been developed. Nanoparticles, due to their small size, adhere to and thus foul the component surfaces. The thermal conductivity cell cleaning has been considered an important factor in design. The present design has been conceptualized to provide a flexible method to easily replace the sample and disassemble the cell to clean the parts. Some of the important design factors that have been considered are: controlled tension of the hot-wire, flexibility in handling and cleaning, and electrical signal wiring connections.

The main design parameters are: (a) material of hot-wire, (b) radius of hot-wire, (c) insulation coating, (d) length of hot-wire, (e) radius of the test sample outer boundary, and (f) length of the sample.

Platinum has been selected as superior hot wire material. It has higher thermal conductivity (TC) compared to the nichrome and tantalum, also used as hot-wires. Along with the material, hot-wire radius is one of the most important parameters for the cell design. Among commercially available sizes, 25.4 and 50.8 \( \mu \text{m} \) radius platinum-wires have been selected for the present application, since smaller 12.5 \( \mu \text{m} \) radii is considered to be too fragile for cleaning and handling of nanofluid samples.

Teflon has been selected as insulating material, as it is highly resistant to chemical reactions, corrosion and stress-cracking at high temperatures. A 50.8 \( \mu \text{m} \) diameter platinum wire with a Teflon insulation coating of 25.4 \( \mu \text{m} \) thickness, manufactured by A-M Systems, Inc., has been used as the hot-wire. Care has been taken to avoid any disruption of the coating during hot-wire mounting.

In our design, the length of the platinum hot-wire was taken as \( L_w = 0.1484 \text{ m} \), based on the 0.139 \text{ m}
To the Data Acquisition System

wire cell outer boundary radius was determined as 0.0028 m, based on Healy et al. [23]. The minimum hot-wire length of hot-wire, determined according to Kierkus et al. criteria [22], for our application data [1]. Based on Healy et al. [23] criteria, the minimum hot-wire outer boundary radius was determined as 0.0028 m, but chosen to be \( R_c = 0.00718 \) m, and the finite length of the sample to be \( L_c = 0.170 \) m. The overall sample volume \( V_c \) after fabrication is calibrated to be 35 ml.

A cross-sectional view of the newly designed hot-wire thermal conductivity apparatus with major mechanical components is shown in Fig. 1. The major assembly components of the apparatus’ cell are: base plate, outer shell, and cell cap with hot-wire. The cell base plate with a threaded hole at the center of the plate (sealed by a Teflon washer) is used for convenient assembling and disassembling the outer shell. The outer shell with 17.4 mm inner diameter acts as the sample test fluid reservoir. The cell cap, designed to slide-fit into the outer shell, is hollow inside. The inner semi-circular hot-wire holder with an alignment ring is soldered at the lower end at an offset. A hot-wire guiding block, sliding tube, tension spring, and spring rod are all aligned at an offset inside the cap. The Teflon-coated platinum wire is indirectly connected to the tension spring via copper wires and a sliding rod, which are aligned with the spring mechanism (i.e., sliding tube, tension spring, spring rod and locking nut). A locking nut, fastened to the spring rod, is mounted on the top of the cap. Two symmetric rectangular cuts in the cap provide an opening for routing of electrical and thermocouple wires. A connector and calibration gauge holder and a wire holder, made of Teflon, are mounted on the top and middle section of the cell cap.

Three thermocouples, mounted on the outer radius of inner semi-circular hot-wire holder at 15º, 45º and 75º angles, along the length of the sample test section, monitor uniformity of the test fluid temperature. The thermocouple tip is bent towards the platinum hot-wire through the holes on the inner semi-circular hot-wire holder.

Two copper wires at the top soldered-joint of the platinum hot-wire are passed symmetrically through a sliding tube. The inner hollow portion of the sliding tube is filled with epoxy to couple the copper wires with the sliding tube. A clearance between the sliding tube and hot-wire guiding block hole ensure near-frictionless motion of the sliding tube. The spring rod is specially shaped to have external threads. A locking nut and spring rod are screwed together like a nut and bolt. The special shaped sliding fit hole avoids turning of the spring rod when the locking nut is turned for adjusting the tension of the hot-wire. The locking nut has been fabricated to a specific weight that is used for calibration of the platinum hot-wire tension, within 50% of its ultimate tensile strength. An inverted L-shaped gauge has been seated on the holder for calibrating the hot-wire tension and guarding the spring rod movement. One of the two copper wires at another soldered joint of the platinum hot-wire is passed through the off-centered hole of the alignment ring, while the other has been guided through a hole in the inner semi-circular hot-wire holder. The copper wires at the alignment ring act as fixed rigid ends of the hot-wire.

The present cell parameters are \( L_c = 0.170 \) m, \( 2R_c = 0.01437 \) m, \( V_c = 35 \) ml, \( L_c/2R_c = 11.83 \). The parameters of the platinum hot-wire with Teflon coating of thickness 25.4 \( \mu \)m are \( L_{w} = 0.1484 \) m,
The controlled tightness of the thin platinum hot-wire is a very important aspect. An innovative solution, to indirectly connect a tension spring to the platinum hot-wire, while maintaining constant tension, has been incorporated. Also, a unique solution to calibrate the platinum hot-wire tension has been developed. This arrangement minimizes the well known and unwanted strain gage effect on hot-wire electrical resistivity, thus decreasing the strain influence on temperature measurement. An extension spring with a low spring constant is calibrated and used for present application as detailed elsewhere [1].

4. Instrumentation and Data Acquisition

A Wheatstone bridge circuit has been employed to measure the resistance change of the hot-wire, with the wire being one of the arms (i.e., resistors) of the bridge. Initially, the bridge is balanced until the voltage output of 10-15 µV is achieved. The bridge balancing is performed within a brief period of time, using a constant, low input voltage of 0.1 V, to minimize heating of the platinum wire during initial bridge balancing. After the bridge circuit is balanced, a constant, input voltage $V_{in}$, at start-time $t = 0$, is applied to heat the wire, thus resulting in unbalancing of the bridge due to the hot-wire’s temperature and thus resistance change. The bridge input $V_{in}$ and output $V_{out}$ voltages are measured using a computerized data acquisition system. A schematic diagram of the Wheatstone bridge circuit used for measurement is shown in Fig. 3. The circuit has been fabricated in such a way that the data acquisition system can be easily connected and disconnected for measurement. The schematic of DAQ system is also shown in Fig. 3.

Six different signals are measured using National Instruments' data acquisition (DAQ) hardware, namely: bridge voltage output; bridge voltage input; hot-wire voltage drop (i.e., voltage drop across platinum wire); and three signals from the thermocouples mounted across the length of the hot-wire cell at the top, middle, and bottom sections. A computer program for acquiring and post-processing the measured data is developed using the LabVIEW® application software.

All measurement parameters are controlled using the LabVIEW® user interface. The input voltage range is configured at 0 – 10 V. The bridge voltage output is triggered after the voltage threshold reaches a value of 1 mV. The voltage output channel can be configured to a maximum sampling rate is 100 Hz, with nominal operation range configured at 0 – 200 mV, which

![Fig. 2: Fabricated transient hot-wire thermal conductivity apparatus](image1)

![Fig. 3: Schematics of electrical circuit with computerized data acquisition system](image2)
provides an overall measurement gain of 100. The bridge voltage output and time are measured and stored simultaneously. Post-processing of the acquired data is then performed in order to calculate the resistance change, temperature change, heat input, and then thermal conductivity of the test fluid.

5. Calibration and Uncertainty Analysis

Two standard base fluids of well-defined thermal conductivity, ethylene glycol and distilled water, have been used for over-all calibration of the apparatus. The voltage change (i.e., bridge voltage output) for all calibration measurements has been acquired at a sampling rate of 50 Hz. For a typical measurement, the resistances of the Wheatstone bridge circuit are measured as: \( R_1 = 2270.6 \, \Omega \), \( R_2 = 2161.1 \, \Omega \), and \( R_3 = 7.715 \, \Omega \), using a four-wire resistance measurement technique. The reference resistance of the platinum wire is determined as \( R_{w0} = 8.106 \, \Omega \) using relevant correlations [1]. The measured length of platinum hot-wire was \( L_w = 0.1484 \, m \). The platinum temperature coefficient of resistance is \( \sigma_{TCR} = Z_{\sigma} / R_{w0} \), where \( R_{w} = 8.22 \, \Omega \) is the resistance measured at 20°C and \( Z_{\sigma} = 0.02652 \, \Omega/°C \) is the slope of the hot-wire resistance with regard to temperature.

Fig. 4 shows a typical hot-wire temperature change versus time for ethylene glycol and water. Analysis of the graph indicates that the linearity of the wire temperature change (in semi-log coordinate system) is obtained one second after the experiment starts. The initial deviation from linearity at later times, with higher temperature differences, can be attributed to on-set of convection heat transfer and finite boundary effects [24, 25]. Under similar testing parameters, the temperature change in time for water is lower compared with that of ethylene glycol, which is attributed to the fact that water, compared to ethylene glycol, has higher thermal conductivity, see Eq. (5).

Thermal conductivity measurements of ethylene glycol and water have been repeated 10 times each, and average values reported. The time range from 1 to 10 s has been determined to be virtually linear, and measurement sub-range from 2 to 6 s is chosen for increased accuracy, see Table 1. The reference temperature \( T_r \), at which the test fluid properties are measured, is evaluated as the process average temperature:

\[
T_r = T_0 + \frac{1}{2} \left[ \Delta T(t_1) + \Delta T(t_2) \right]
\]

(10)

Where, \( T_0 \) is the initial temperature of the fluid (determined using thermocouples mounted within the hot-wire cell); and, \( \Delta T(t_1) \) and \( \Delta T(t_2) \) are measured temperature increases at times \( t_1 \) and \( t_2 \), respectively [23]. The standard thermal conductivity values of ethylene glycol and water are obtained from a standard engineering reference.

Ethylene glycol with 99.9 % purity and distilled water, have been used as standard test fluids for over-all calibration of the new HWTC apparatus. The mean reference

<table>
<thead>
<tr>
<th>Fluid</th>
<th>Reference [W/m°C]</th>
<th>Measured [W/m°C]</th>
<th>Bias Error</th>
<th>Precision Error (95 %)</th>
<th>Uncertainty in Repeatability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ethylene Glycol (32.5 °C)</td>
<td>0.254</td>
<td>0.253</td>
<td>- 0.395 %</td>
<td>2.03 %</td>
<td>2.06 %</td>
</tr>
<tr>
<td>Distilled water (~ 26 °C)</td>
<td>0.612</td>
<td>0.619</td>
<td>1.2 %</td>
<td>2.23 %</td>
<td>2.52 %</td>
</tr>
</tbody>
</table>

NOTE the required linearity range from 1-10 s, reduced from 2 to 6 s measurement range for increased accuracy.

Table 1: Measurement uncertainties and repeatability errors of measured thermal conductivity

Fig. 4: Hot-wire temperature change against time (in logarithmic scale) for ethylene glycol and distilled water.

Fig. 4: Hot-wire temperature change against time (in logarithmic scale) for ethylene glycol and distilled water.
temperature of ethylene glycol, using Eq. (10), was measured 32.5 °C, with the respective reference thermal conductivity of ethylene glycol 0.254 W/m°C, while the corresponding values for distilled water were 26.0 °C and 0.612 W/m°C, respectively.

A detailed calibration and the measurement uncertainty analysis have been performed; see Table 1, and elsewhere [1].

6. Nanofluid Thermal Conductivity

Nanofluids, of 1 % volumetric nanoparticle concentration, have been prepared by the two-step method [7], using 35 nm mean-size copper nanoparticles (manufactured by Nanotechnologies, Inc.) with ethylene glycol and distilled water as the base fluids. A digital analytic balance of 0.0001 g resolution has been used for measurements. The physical stabilization method of ultra-sonication has been applied to the nanofluids, by placing the nanofluid container in an ultrasonic bath for about 10 hours. The nanofluids have been measured using the calibrated transient hot-wire thermal conductivity apparatus described above. The mean nanofluid thermal conductivity of 1 % volumetric copper-nanoparticle concentration in ethylene glycol, \( k_{f,EG}^n \), is measured as 0.2866 W/m°C, and the corresponding relative thermal conductivity, \( k_{f,EG}^n / k_{f,EG} \), which yields about 13 % enhancement with regard to the \( k_{f,EG} = 0.254 \) W/m°C, the base-fluid reference thermal conductivity of ethylene glycol, see Fig. 5.

Similarly, the mean nanofluid thermal conductivity of 1 % volumetric copper-nanoparticle concentration in water, \( k_{f,WR}^n \), is measured as 0.7107 W/m°C, and the corresponding relative thermal conductivity, \( k_{f,WR}^n / k_{f,WR} \), which yields about 16 % enhancement with regard to the \( k_{f,WR} = 0.613 \) W/m°C, the base-fluid reference thermal conductivity of distilled water.

7. Conclusion

An apparatus based on the single, transient hot-wire method has been developed, designed and fabricated with main objective to measure thermal conductivity of fluids, polymer solution, nanofluids and poly-nanofluids [1,2,3]. The goal was to reduce the overall test sample volume for nanofluids, while maintaining the precision and accuracy of the apparatus. The new apparatus employs innovative solutions for easy calibration of controlled platinum, hot-wire tension, and thus minimizing the strain influence on temperature measurement (i.e., minimizing the well-known and unwanted “strain-gage effect” on Pt-wire electrical resistivity); measurement of Pt-wire voltage drop independently from power wiring (four wires); and an effective off-centered mechanical design to minimize the fluid sample size (about 35 mL). Data acquisition hardware and LabVIEW® application software are optimized to minimize signal noise and enhance acquisition and processing of useful data.

The bias measurement error, based on calibration with distilled water and ethylene glycol, has been found to be within 1.5 %, and precision, i.e., repeatability error within 2.5 %. The total uncertainty in measured thermal conductivity has been estimated to be within 3.5 % at 95 % confidence probability. Two nanofluids of 1 % volumetric of 35 nm-size copper-nanoparticle concentration in ethylene glycol and in water, were prepared and their thermal conductivity measured using the new apparatus. The mean enhancement in thermal conductivity of nanofluids with copper nanoparticles in ethylene glycol and in water, were measured as 13 % and 16 %, respectively. The developed apparatus has proved to be efficient and with potential for further improvement.

![Fig. 5: Nanofluid thermal conductivity measurements of 1 vol % of 35 nm copper nanoparticles in ethylene glycol](image-url)
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