Unleashing
Error or Uncertainty Analysis
of Measurement Results

"Complex and Ambiguous,
BUT…logical and possible!"

Prof. M. Kostic
Mechanical Engineering
NORTHERN ILLINOIS UNIVERSITY

At NASA/Cleveland …

This is <u>my first year</u> at NASA and in Cleveland, and everything is **beyond my expectations!**

Make it effective and short ...

Dr. Joseph (Joe) Prahl advised me
to make presentation effective
by stating only

<u>the most important</u> things …?!

www.kostic.niu.edu

What is <u>the most important</u> ...?

... **<u>Nothing is</u>** as important
as we think it is

…because, **<u>nothing is</u>** what we think it is!

*** Be aware of complexity but make it simple! ***

www.kostic.niu.edu

…so we may conclude this presentation now...

… and answer your Questions ?

if any

Just kidding !

Uncertainty of Measurements ...

- **not** a **deterministic** (exact), **but** rather **holistic and probabilistic** in nature.
- **It's complex and ambiguous:**
  - not only **what is measured** (**any operator**)
  - but **what contributes to errors and uncertainty**, the latter being **open ended**) (**'skilled' researcher**)

Almost impossible task ...

- **to resolve <u>existing confusion</u>**
- **to embrace <u>the very concept</u> of uncertainty**
- **to provide <u>effective guidelines</u>:**
  - account for the **<u>most contributing sources</u>** of errors (which is **<u>important and possible</u>!**)
  - accounting for **<u>all sources of errors</u>** is **<u>not necessary</u>** (and also **<u>impossible</u>).**

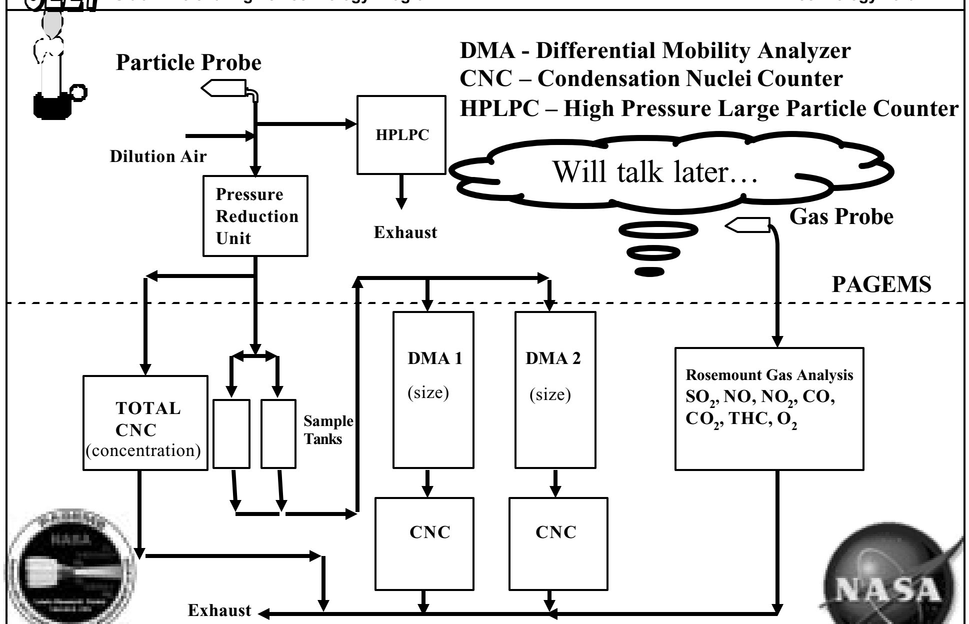
Only...

... if the **<u>uncertainties</u>** of measurements are **<u>quantified</u>**, the measurements may be **<u>judged</u>**

- if they are **<u>adequate</u>** for intended purpose i.e. **<u>useful</u>**, or
- **<u>consistent</u>** with other similar results.

Evaluation/**Quantification** of Uncertainties ...

- **components' identification** (very critical)
- **quantification** of all standard uncertainties (experience)
- use **the law of propagation** of uncertainties
- reduce all components to the **same probability** (of course same unites) and **combine** them using the **R**oot-**S**um-of-**S**quares (**RSS**) **rule**
- **Expend** the combined uncertainty to desired confidence level
- Finally, **report** relevant details (also **important** to include what is **not** accounted for)



PAGEMS-Particulate and Gaseous Emissions Measurement System

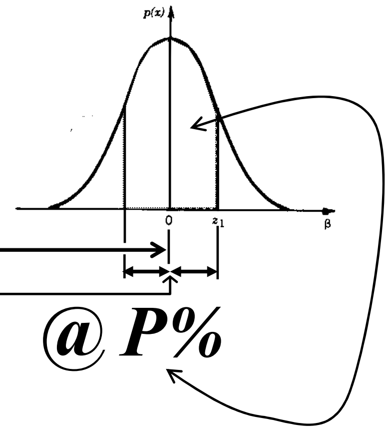
Probability and Statistics
<u>Variations</u> are due to:

- **Measurement System:**
  Resolution and Repeatability
- **Meas. Procedure:**
  Repeatability
- **Measured Variable:**
  Temporal & Spatial Variations

Statistical Measurement Theory

- **Sample - a 'selected set' of 'population'**
- **Measurand - measured variable** (x)
- **True** x' **vs. mean** x_{mean} **value**

Mean Value and Uncertainty



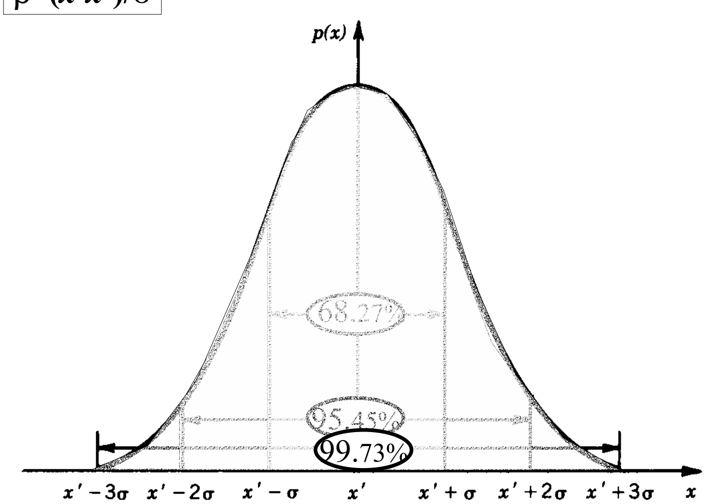
$$x' = x_{mean} \pm u_x \quad @\, P\%$$

x_{mean} is a *P%* probable estimate of x' with uncertainty u_x

Normal-Gaussian distribution

$$P(-z_1 \le \beta \le z_1) = \frac{1}{(2\pi)^{1/2}} \int_{-z_1}^{z_1} e^{-\beta^2/2}\, d\beta = 2\left[\frac{1}{(2\pi)^{1/2}} \int_{0}^{z_1} e^{-\beta^2/2}\, d\beta\right]$$

$\beta = (x - x')/\sigma$



Standard Deviation of the Mean

$$S_{\bar{x}} = \frac{S_x}{N^{1/2}} \quad (4.16)$$

$$\bar{x} \pm t_{\nu,P} S_{\bar{x}} \quad (P\%)$$

$$x' = \bar{x} \pm t_{\nu,P} S_{\bar{x}} \quad (P\%) \quad (4.17)$$

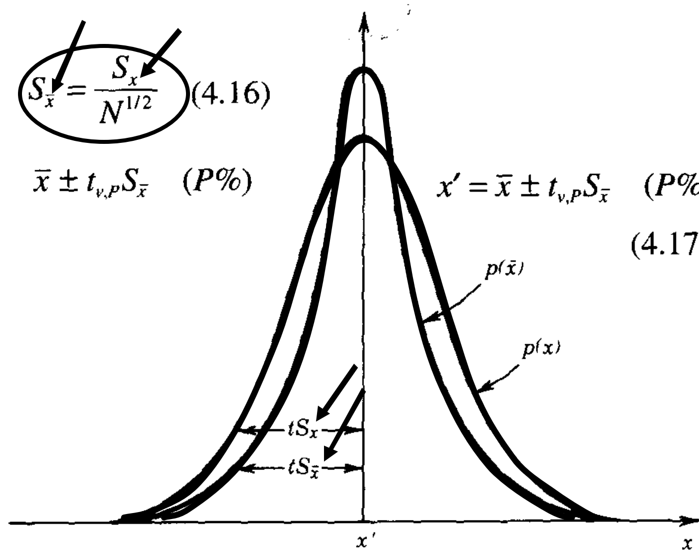
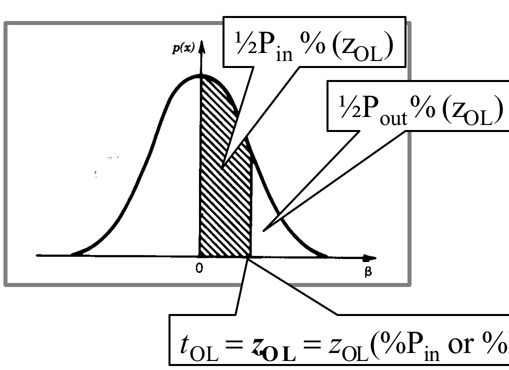


FIGURE 4.6 Relationships between S_x and a distribution of x and between $S_{\bar{x}}$ and the true value x'.

Data Outlier



$t_{OL} = \boldsymbol{z_{OL}} = z_{OL}(\%P_{in} \text{ or } \%P_{out})$

Usually $\mathbf{z_{OL}= 3}$ or
$\mathbf{z_{OL}=}$
$\mathbf{z_{OL}}(\frac{1}{2} P_{out}= 0.5-\frac{1}{2}P_{in}=\mathbf{0.1/N})$
if number of data N is large.
(For $\mathbf{P_{out}=1\%}$, $z_{OL}=\mathbf{2.33}$)

Keep data if within ± $\mathbf{z_{OL}}$
otherwise REJECT DATA
as Outliers

Required #of Measurements

Mean precision interval : $CI = 2d = \pm u = \pm d = \pm t_{\nu,\%P} \frac{S_x}{\sqrt{N}}$; then..

$N = \left(\frac{t_{\nu,\%P} S_x}{d} \right)^2$; since $\nu = N - (m+1)\big|_{m=0} = N - 1$

the calculatio n procedure is iterative (unless $N \to \infty$, too large)

www.kostic.niu.edu

Uncertainty/Error Analysis

Errors (e = x-x' ≅ x-x_{avg}= d, **also B, P, S≅σ;**
do NOT be confused, see NIST Guide):

Bias (B), Precision (P), also Standard (S or σ)

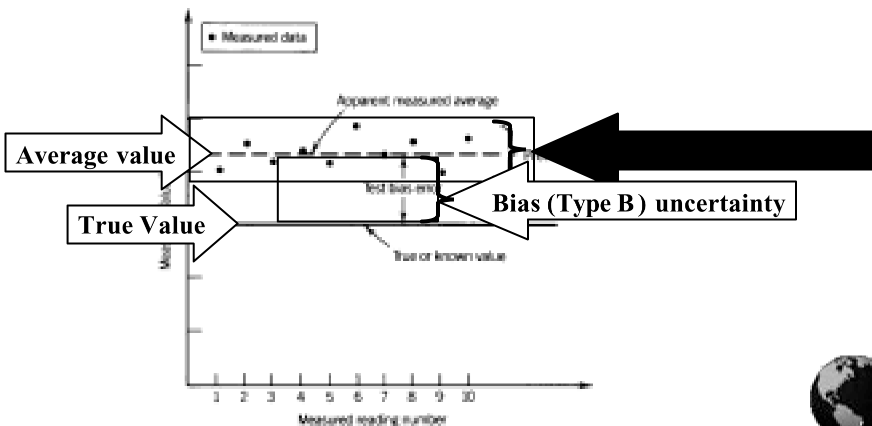
Uncertainty (u) **is the range of errors (e, B, P, S) at corresponding Probability (%P)**

Remember: **$u = d_{\%P} = t_{\nu,\%P}S$ (@ %P); z=t=d/S**

www.kostic.niu.edu

Bias and Precision Errors/Uncertainties

FIGURE 1.9 Effects of precision and bias errors on calibration readings.



Type A Uncertainties ...

due to <u>random</u> errors (also known as <u>precision</u>, or <u>statistical</u> uncertainties), which could be quantified using available <u>statistical methods</u> on a set of <u>repeated measurements</u> under apparently the same (or very similar) conditions.

Type B Uncertainties ...

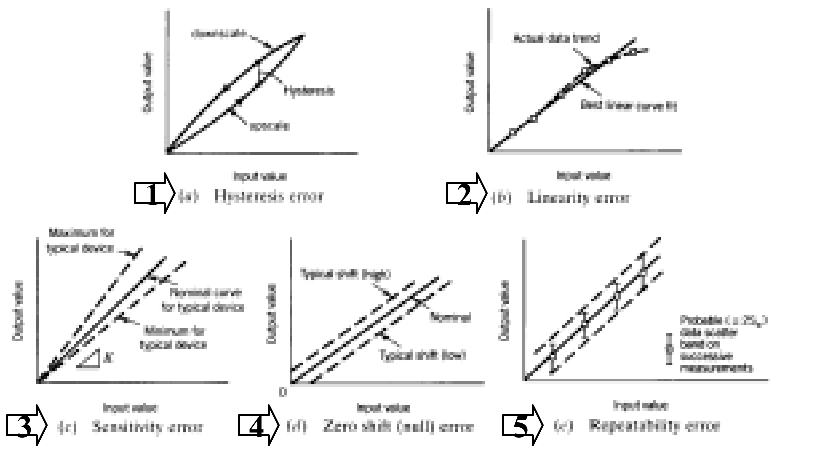
due to other than statistical errors associated to instrumentation and measurement procedure used (often unspecified, also known as biased or systematic errors). They are evaluated using scientific judgment and previous related experience and calibration, including instrumentation specifications, other outside sources and references (all-time history), and judgmental assumptions.

Systematic errors ...

… and specified biases if quantified (like after careful calibration) are no longer uncertainties, since they may be corrected, but only unspecified biases (due to unknown/random nature) are inherited uncertainties of instrumentation and similar, previous measurements.

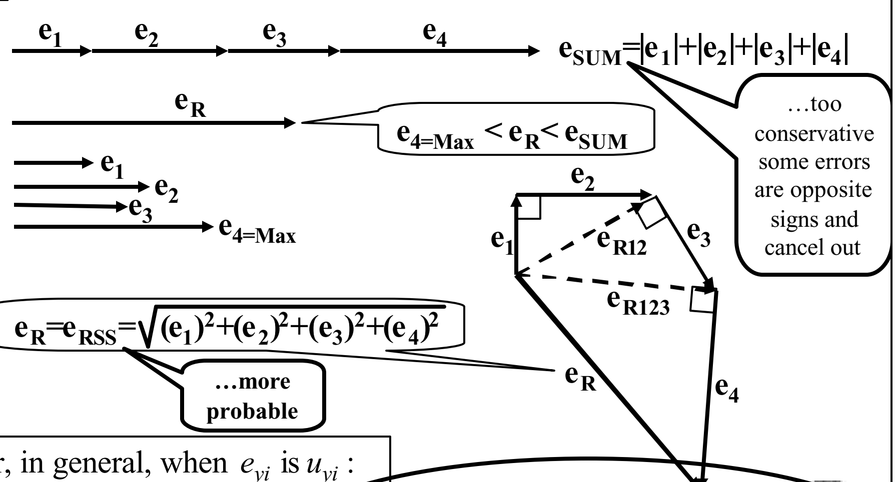
Different Instrument Errors

FIGURE 1.10 Examples of elements of instrument error.
(a) Hysteresis error. (b) Linearity error. (c) Sensitivity error.
(d) Zero shift (null) error. (e) Repeatability error.



$$u_y = \sqrt{u_{y1}^2 + u_{y2}^2 + \ldots + u_{yL}^2} = \sqrt{\sum_{i=1}^{L} u_{yi}^2}$$; i.e. RSS (Root – Sum – Square) procedure

RSS-Errors Summing



…or, in general, when e_{yi} is u_{yi} :

$$u_y = \sqrt{u_{y1}^2 + u_{y2}^2 + \ldots + u_{yL}^2} = \sqrt{\sum_{i=1}^{L} u_{yi}^2}$$; i.e. RSS (Root – Sum – Square) procedure

Error Sources

1. Calibration Error Source Group

   Table 5.1 - (B or P)$_{1j}$

2. Data Acquisition Error Source Group

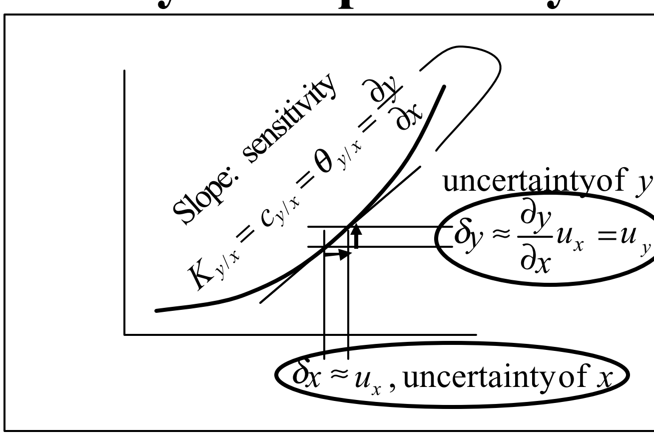
   Table 5.2 - (B or P)$_{2j}$

3. Data Reduction Error Source Group

   Table 5.3 - (B or P)$_{3j}$

It is **not important which group** an error is assigned to, **as long as it is accounted for**.
The groups and their items are **for convenience only**.

Sensitivity or Dependency Rate



If one variable "y" depends on another "x," then a small change of x, i.e $\delta x \approx u_x$ (error, uncertainty) will propagate as error of y, i.e. $\delta y \approx u_y$.
Using the partial derivative, i.e. the rate of dependancy, or sensitivity:

$$\delta y \approx \frac{\partial y}{\partial x}\delta x \approx \frac{\partial y}{\partial x}u_x = (\theta_{y/x})u_x = u_y \text{ ; where sensitivity } K_{y/x} = c_{y/x} = \theta_{y/x} = \frac{\partial y}{\partial x}$$

Error Propagation (Combined Uncertainty)

If one variable y depends on another x, then a small change of x, i.e $\delta x \approx u_x$ (error, uncertainty) will propagate as error of y, i.e. $\delta y \approx u_y$:

$$\delta y \approx \frac{\partial y}{\partial x}\delta x \approx \frac{\partial y}{\partial x} u_x = (\theta_{y/x}) u_x = u_y \text{ ; where sensitivity } \theta_{y/x} = \frac{\partial y}{\partial x}$$

For a multyfunction variable $y = y(x_1, x_2, \dots x_i, \dots x_L)$:

$$u_y = \sqrt{\sum_{i=1}^{L} \left(\theta_{y/xi} u_{xi}\right)^2} = \sqrt{\sum_{i=1}^{L} u_{yi}^2} \text{ ; i.e. RSS (Root – Sum – Square) procedure}$$

$$\text{where } u_{yi} = \begin{Bmatrix} u_{yi} \text{ a known elemental error} \\ \text{or } u_{yi} = \left(\frac{\partial y}{\partial x_i}\right) u_{xi} = (\theta_{yi/x}) u_{xi} \end{Bmatrix}$$

Design-stage Uncertainty
(Instrument errors only)

Design - stage error/uncertainty

$$u_d = \sqrt{u_0^2 + u_c^2}$$

Interpolation error

$$u_0 = \pm \tfrac{1}{2} \text{ resolution}$$

Instrument error

$$u_c \quad (by\ calibration)$$

Advanced-stage Uncertainty

(Instrument and measurement errors)

N^{th} order uncertainty

$$u_N = \sqrt{u_d^2 + \sum_{i=1}^{N} u_i^2}$$

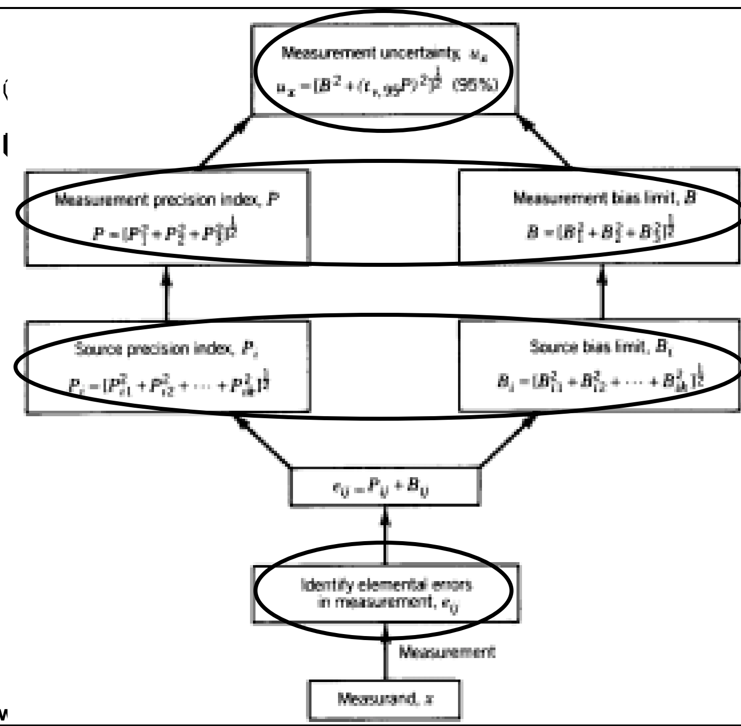
Zero - and design order uncertainties

$$u_0 = \pm \tfrac{1}{2} \text{ resolution}; \; u_d = \sqrt{u_0^2 + u_c^2}$$

First - order uncertainty

$$u_1 \geq u_0, u_d$$

Multiple-measurement Uncertainty



Error Summation/Propagation
(Expanded Combined Uncertainty)

$$u_R = \sqrt{B_R^{\,2} + (t_{\nu_R,\%P} P_R)^2}\,,\ \text{where}\ :$$

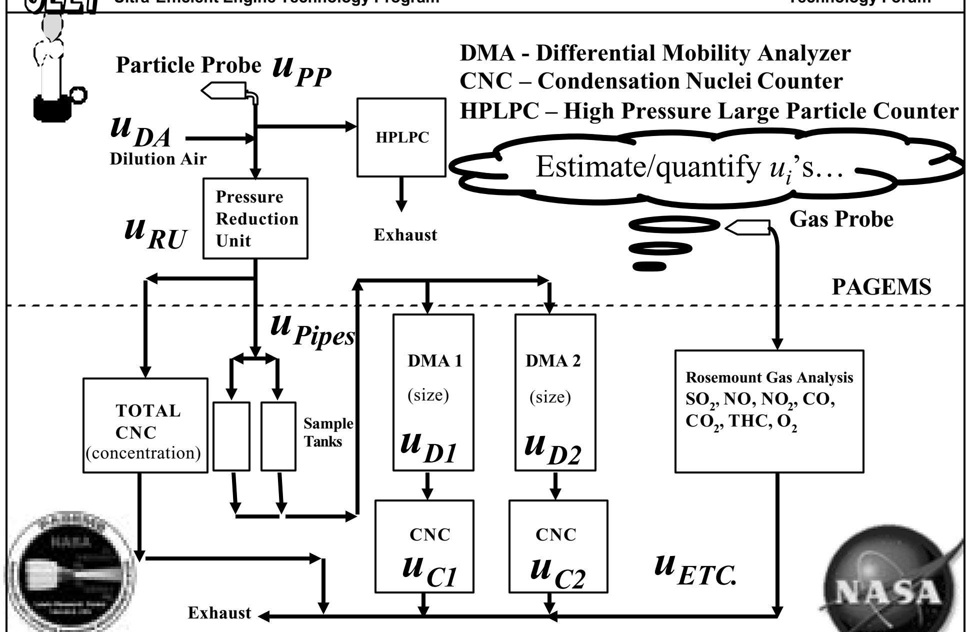
$$B_R = \sqrt{\sum_i B_i^{\,2}}\quad \text{and}\quad P_R = \sqrt{\sum_i P_i^{\,2}}\ ;\ \text{also}\ :$$

$$\nu_R = \frac{\left(\sum_i P_i^{\,2}\right)^2}{\sum_i \frac{P_i^{\,4}}{\nu_i}}\ ;\quad \nu_i = N_i - 1,\ \#\,\text{of degree of freedom}$$

Note, it could be $P_i = P_{yi} = \theta_{yi/xi} P_{xi}$

or, it could be $B_i = B_{yi} = \theta_{yi/xi} B_{xi}$

Ultra-Efficient Engine Technology Program — Technology Forum

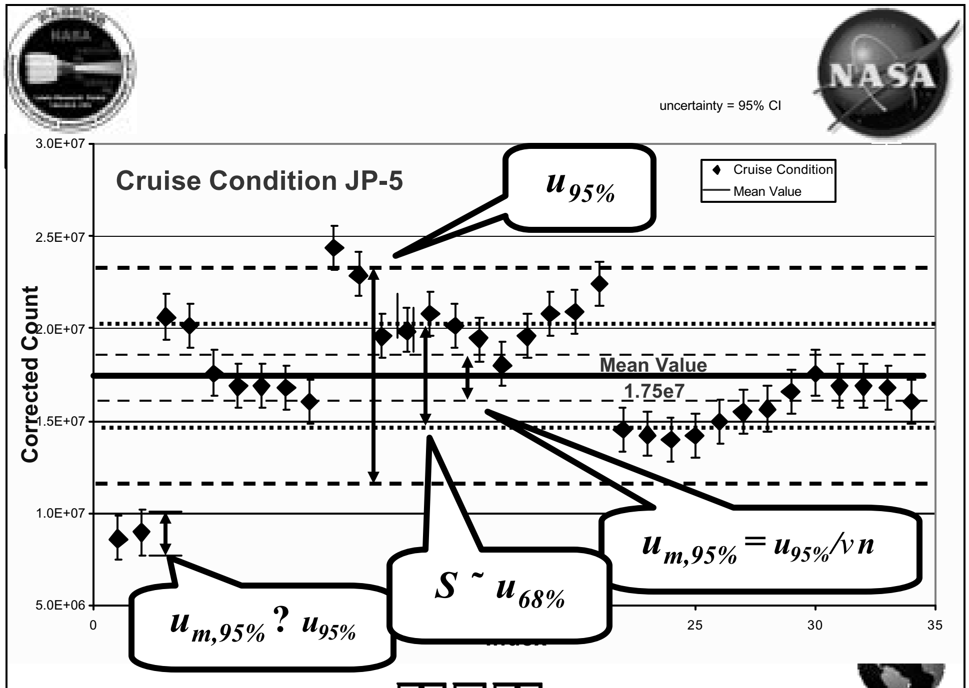


PAGEMS-Particulate and Gaseous Emissions Measurement System

Row #	Current nomencl.	Excel functions and formulas	Excel Descriptive Statistics' items
1	x_m	AVERAGE(Data)	Mean (average)
2	S_{xm}	STDEV(Data)/SQRT(COUNT(Data))	Standard Error (of mean)
3		MEDIAN(Data)	Median (mid-way of data)
4		MODE(Data)	Mode (most frequent data)
5	S_x	STDEV(Data)	Standard Deviation (of data sample)
6	S_x^2	VAR(Data)	Sample Variance
7		KURT(Data)	Kurtosis (peakedness)
8		SKEW(Data)	Skewness (asymmetry)

continue …

Row #	Current nomencl.	Excel functions and formulas	Excel Descriptive Statistics' items
		…..continue …	
9		MAX(Data)-MIN(Data)	Range
10		MIN(Data)	Minimum
11		MAX(Data)	Maximum
12		SUM(Data)	Sum
13	n	COUNT(Data)	Count (number of data)
14		LARGE(Data, i_th)	Largest(i_th=1)
15		SMALL(Data, $i_{\_th}$)	Smallest(i_th=1)
16	$u_{?,P\%}=t_{?,P\%}\cdot S_{xm}$	TINV(1-Probability, COUNT-1)*Standard_Error	Uncertainty at Confidence Level (for 95.0%=Probability)
17	$P_{t,?}$	1-TDIST(t, COUNT-1,2)	Probability (Double-Insid[e] for given deviation *t*



Conclusion…

It is important to <u>clarify again</u>, that …

<u>only</u> the <u>uncertainty</u>,
i.e. a <u>deviation/error range</u> (or interval)
from a reported measurement result,
<u>with</u> corresponding <u>probability</u>,
may be evaluated …

Conclusion…

… but it is not possible to obtain
a perfect (error-free) measurement,
nor it is possible to estimate an uncertainty
with 100% probability
(absolute certainty is impossible too).

Conclusion…

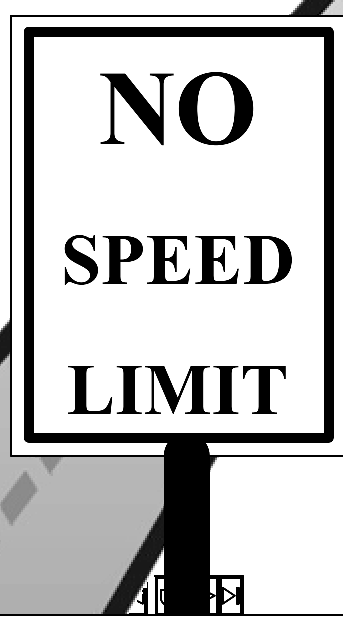
However,
under well-controlled conditions and
well-understood measurement process and
procedure, it is possible to minimize and
reasonably well (at high probability)
estimate the uncertainties
of measured quantities and
the final measurement result.

Who does the future belongs to ?

?

No Limits …

The Future Belongs To…



… Whoever Gets There First

REFERENCES:

[1] Guidelines for Evaluating and Expressing the Uncertainty of NIST Measurement Results, NIST Technical Note 1297, 1994 Edition http://physics.nist.gov/Pubs/guidelines/TN1297/tn1297s.pdf , July 2003.

[2] Galleries of Probability Distributions: *NIST/SEMATECH e-Handbook of Statistical Methods*, http://www.itl.nist.gov/div898/handbook/eda/section3/eda366.htm , July 2003.

[3] R.H. Dieck, "Measurement Accuracy" a chapter in "The Measurement, Instrumentation and Sensors Handbook" *(J.G. Webster, Editor-in-Chief)*, ISBN: 0-8493-8347-1, CRC Press, 1999.

[4] *NIST/SEMATECH e-Handbook of Statistical Methods*, http://www.itl.nist.gov/div898/handbook/, July 2003.

[5] Tables for Probability Distributions: *NIST/SEMATECH e-Handbook of Statistical Methods*, http://www.itl.nist.gov/div898/handbook/eda/section3/eda367.htm , July 2003.

[6] R.S. Figliola, and D.E. Beasley, *Theory and Design for Mechanical Measurements* - 3rd Edition, Wiley, 2000.

Acknowledgements/Thanks

Dr. Chi-Ming Lee
Dr. Paul F. Penko
Stephen H. Culler
Terry M. Sanders
Alexia Finotello

And my office-mates

Dr. Eric R. McFarland
Dr. Rod V. Chima

www.kostic.niu.edu

© MMIII* by M. Kostic

Contact Information:

Open for collaboration (and grants…)
(‘obsessed’ with problem-troubleshooting!)

mailto:kostic@niu.edu

or on the Web:

www.kostic.niu.edu

http://prof.mkostic.com