

## Sampling and Aliasing: An Interactive and On-Line Virtual Experiment

*“What we ‘see’ is not what it is!”*

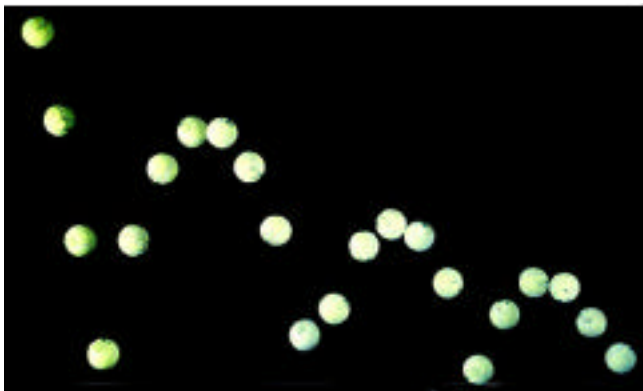
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### Abstract

An interactive virtual instrument is further developed for use in class and on-line over the Internet, to simulate sampling and aliasing phenomena, an essential concept in many science and engineering courses, including physics, dynamics, control, measurement methods, and digital signal processing. A stroboscope-mechanical system is used to further visualize concept and show striking similarity with electronic data gathering and to demonstrate aliasing. While students learn procedures to calculate the system frequency response, many do not fully understand how and why a real-time signal output is distorted. As the aliasing phenomena are added to the measurement challenge, the students become confused and discouraged to get involved and fully comprehend these important physical processes. The virtual instrument allows users to easily experiment (or play) with different signals and instrument characteristics, and interactively see the relevant outcomes, the system frequency response and aliasing if present, along with input and output signals, both qualitatively (visually/graphically) and quantitatively (numerically). The interactive simulator should stimulate users' curiosity and accelerate learning by active, “what-if” inquiry and experimentations, and thus, enhance their experience and comprehension. Each and every one of these new tools, when designed well and used creatively, may qualitatively enhance the learning environment.



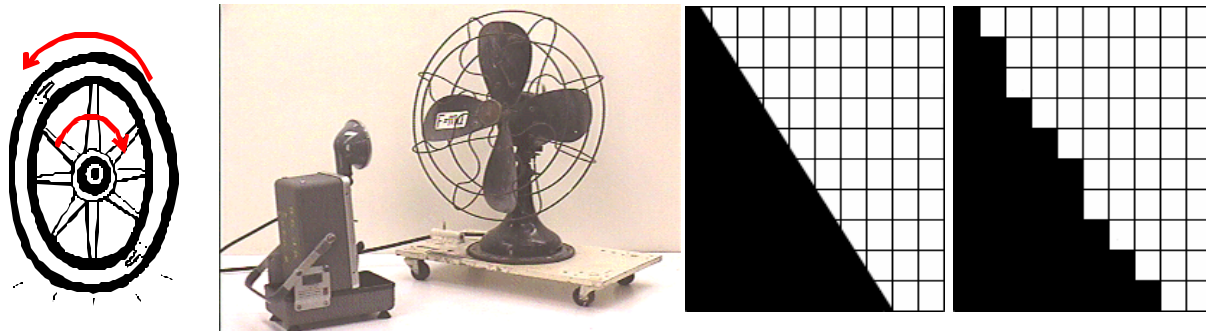
**Figure 1:** Discrete sampling of the motion of a bouncing ball by stroboscope light on a single picture frame (“supper-fast” photography).

### Introduction

What are “*sampling*” and “*aliasing*” and why they are important? In science and engineering, sampling means “discrete observing” or measuring, or probing with a certain “sampling” rate or frequency, like with a stroboscope blinking light, see Figure 1. Aliasing happens when an analog object, signal or data is represented (measured or “seen”) by a discrete system, i.e. in a discrete domain or a grid. Used in the context of processing digitized waveform signals (e.g. audio) and images (e.g. video), aliasing describes the effect of under-

sampling during discretization which can generate a false (apparent) low frequency for signals,

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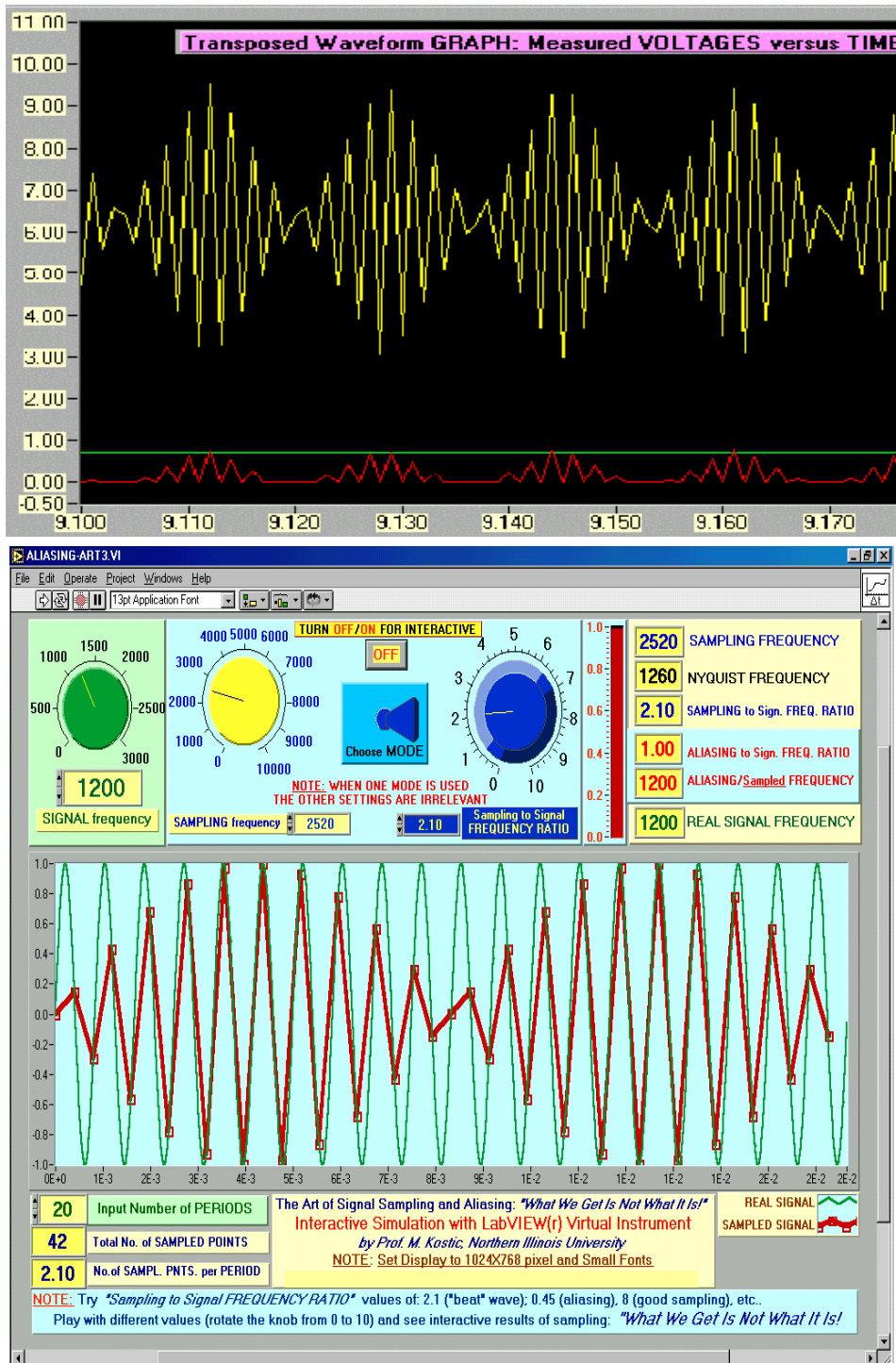
**Figure 2:** The aliasing examples: “wagon wheel illusion,” appearance that a spoke wheel rotates backward while it moves forward (left); a stroboscope experiment to “freeze” rotating fan blades or to make them appear to move slowly forward or backward (center); and appearance of staircase steps along edges (jaggies) in an image (right).

or staircase steps along edges (jaggies) in images, see Figures 2 & 3.

When the analog data/information is converted to discrete or digital, certain problems arise. In addition to missing the details (fidelity) in-between the successive sampled data, the sampled data may “fold-back” and give us the wrong impression (illusion) that something exists that actually does not. That is why it is very important, but not always easy, to fully understand sampling and aliasing. Aliasing may appear in many different processes and in different forms, since real processes are observed or measured as diverse and complex signals with real instruments that have rather complex properties, and known and unknown shortcomings, related to the task they are supposed to accomplish. Several characteristic real-life examples are depicted and described in Figure 2: the “wagon wheel illusion,” an appearance that a poked wheel rotates backward while it moves forward under certain combinations of rotation and lighting conditions (sometimes seen in movies and on expressways); or an appearance of fan blades to be stationary, moving slowly forward or even backward when lighted with certain blinking light frequencies by a stroboscope; or an unusual interference fringes on an image, or appearance of staircase steps along edges (jaggies) in an image when we know that the edges are smooth. Each and every of these and other false appearances of something that is not, is due to limitations and interference of a perceiving or measurement system and perceived or measured signal, as effectively illustrated in Figure 3, with the developed virtual instrument.

## The Challenge

Learning is a challenging intellectual process, and new technologies have tremendous potential to make an immense difference with its interactive (computational) and multimedia features. When a user/learner is in “the driver seat,” the virtual-reality, if well designed and creatively used, may have several advantages over the reality itself – remember, “*I hear...and I forget, I see...and I remember, BUT, I do...and I understand!*” That is why kids love video games, like TV, but hate old-fashioned lectures. For many, since we are still in the beginning of the information revolution, it is hard to comprehend that interactive/computational simulations will make another revolution in the 21st century in many areas, the way steam power or electricity made industrial revolution in previous centuries.



**Figure 3:** More illusions: if the sampling to signal frequency ratio is close to 2 (i.e. 2.10), the sampled simple periodic signal (green/thin) will appear as a very peculiar, so called "beat" wave shape (simulated red/thick at the bottom), similar to one obtained in real measurement in the lab (top).

In order to better understand and utilize different phenomena and processes, which are occurring in nature, we have to detect, “sense,” or measure them somehow, and then interpret them properly. Since there is much more out there than what our eyes could see, what our ears could hear, and what our other senses could feel, we creatively use different measurement sensors and systems to “see, hear, and feel” for us. However, we should be aware that every real (measurement) system has its limitations in trustworthiness (fidelity) and ranges, very much like our own senses.

In addition to static sensitivity (proportionality between the output and input), the physical (analog) systems have their peculiar dynamic characteristics, namely, in what proportion and how quickly they could react to a (fast) changing input. This is usually represented with well-known parameters, i.e. the output-to-input magnitude ratio and delay or phase lag, commonly called the system frequency response, which is, for a given input and environmental conditions, solely dependent on a measurement system’s design and physical characteristics.

Most of today’s measurement systems, in addition to continuously sensing the input and reacting on it with its own output (like in analog systems), they also probe or sample the measured continuous (analog) input in discrete instances with a certain (limited) probing or sampling rate and represent the output in digital form, i.e. discrete and/or digital domain. With a system discrete probing or sampling at a finite rate or frequency, at least two more fundamental problems are developed: First, in-between the two successive sampling or probing, we are missing the input signal or phenomenon completely. Second, even if we are not interested in those high-frequency signal components, the problem is that they may “fold-back” and form a characteristic “pattern,” thus falsely appearing as low-frequency components that do not exist in reality at all, thus trick us (will be explained later). Such phenomena are well-known as aliasing, which is actually an illusion. It is much more than what is learned in basic science and measurements courses, still we have to simplify it in order to explain and understand it, which is a challenging objective.

Thus, every discovered and measured natural phenomenon is only as authentic as our senses or instruments were able to “see” or detect it. It is up to our ingenuity and creativity, or intelligence, as to how to interpret it. But we must always be aware that there is more than to what our “eyes” or instruments could see (we are always missing something, some fine details), and even more importantly, that what we “see” might be an illusion (aliasing) of something that is quite different from its appearance. The bottom line, “*What we ‘see’ is not what it is!*” The developed interactive simulation instrument could effectively demonstrate, visualize and quantify these critical sampling and aliasing phenomena.

## **The Virtual Instrument**

An interactive virtual instrument is further developed for use in class and on-line over the Internet, to simulate sampling and aliasing phenomena<sup>[1]</sup>. To interactively sample or virtually measure (evaluate and plot) a signal versus sampling time, the appropriate simulation procedure is developed as a LabVIEW<sup>®</sup> virtual instrument, to measure a “real-like” signal of arbitrary frequency ( $f$ ), with any sampling frequency ( $f_s$ ), any instrument’s natural frequency ( $f_n$ ), and damping ratio ( $z$ ), or alternatively with the given sampling-to-signal frequency ratio ( $f_s/f$ ), given magnitude ( $M$ ), i.e. measured or sampled-to-signal (output-to-input) amplitude ratio, and phase

shift or phase lag ( $\mathbf{f}$ ). In addition to real measurements, LabVIEW<sup>®</sup> is a very effective application software for interactive what-if simulation in education. It has an advantage to further enhance the simulation applications due to its powerful features and an appealing, real-like “front panel” interface<sup>[2]</sup>.

The virtual instrument modeling is based on the following constraints:

$$(Eq.1): f_M = f_a = \begin{cases} f, & \text{if } 0 \leq f \leq f_{Nyq} \\ f - n_{even}f_{Nyq}, & \text{if } 0 \leq f - n_{even}f_{Nyq} \leq f_{Nyq}, n_{even} = \text{even positive integer} \\ (n_{odd} + 1)f_{Nyq} - f, & \text{if } 0 \leq (n_{odd} + 1)f_{Nyq} - f \leq f_{Nyq}, n_{odd} = \text{odd positive integer} \end{cases}$$

$$(Eq.2): \mathbf{f} = \tan^{-1} \frac{2z \frac{f}{f_n}}{1 - \left(\frac{f}{f_n}\right)^2}; \text{ where } 0 \leq \mathbf{f} \leq \mathbf{p} [\text{rad}]$$

$$(Eq.3): M = \frac{A_M}{A} = \frac{1}{\sqrt{\left[1 - \left(\frac{f}{f_n}\right)^2\right]^2 + \left(2z \frac{f}{f_n}\right)^2}}$$

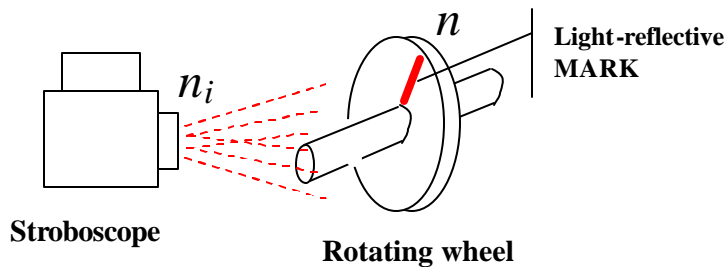
In the above Equations,  $f_M (= f_a)$ , is the measured-output (or aliasing) frequency;  $f$  is the real signal input frequency;  $f_{Nyq} (= 0.5f_s)$  is the Nyquist frequency;  $f_s$  is the sampling frequency;  $f_n$  is the instrument natural frequency;  $\mathbf{f}$  is the phase shift or lag;  $z$  is the damping ratio; and  $M$  is the output-to-input magnitude ratio.

## Art of Measurement of Rotation or Frequency with a Stroboscope:

### *The Strobe Equation*

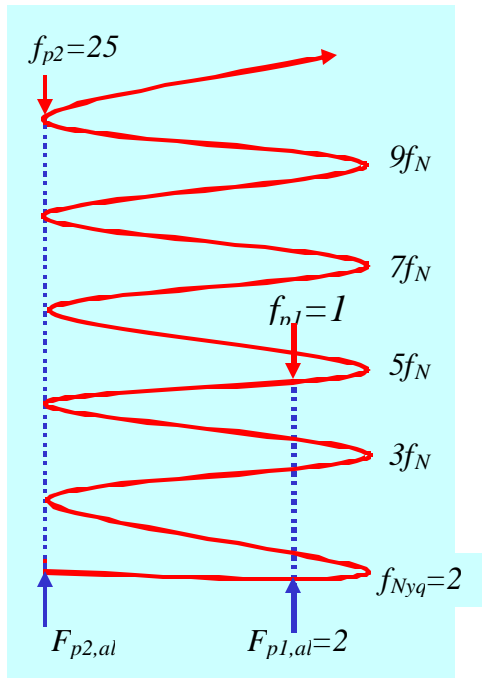
The physics and concept of data sampling and aliasing are the most vivid in real-life physical or mechanical world as perceived by our eye (“*The seeing is believing*”). The concept of sampling is very well demonstrated by measuring angular speed of a rotating wheel in a dark room with a stroboscope. A reflective mark on a rotating wheel, as in Figure 4 for example, will be sampled (seen) when the strobe light fires. If the strobe firing (or blinking) frequency is the same as the wheel's rotational frequency, the reflective wheel's mark will be seen at exactly the same position, and it will appear that the wheel is stationary, does not rotate at all. The same will appear if the wheel rotates at any integer multiple of the strobe light frequency, since the reflective mark will be caught at the same position after that integer multiple of revolutions. These cases correspond to the zero aliasing frequency, i.e. they correspond to the left end of the “folding” diagram, like  $f_2$  on Figure 5. If the wheel rotates somewhat faster or slower, it will appear that the wheel rotates very slowly in a forward or even a backward direction, respectively, which is clearly indicative on the folding diagram in Figure 5. To see the ‘full’ wheel's rotation, the sampling or strobe frequency has to be much bigger than the wheel's rotational frequency. Increasing the strobe frequency will fill up the dark room with ‘continuous-like’ light and the real signal (wheel's rotation) will appear ‘continuously visible.’ A signal sampling is like probing the position of the rotating wheel in the dark with a strobe's intermittent light. If the “sampling” strobe frequency is slow, we may miss a lot of the wheel's rotation “in the dark,” and may wrongly conclude that the wheel rotates much slower or even in the wrong direction, due to aliasing.

Any signal with higher than the Nyquist frequency (equal to a half of the sampling frequency) will appear (or 'fold back') as a smaller aliasing signal frequency, according to the frequency 'folding' diagram, see Figure 5. It will appear as if the signal frequency is much smaller. To avoid this misrepresentation, we have to filter the signal harmonics higher than the Nyquist frequency or to sample at different sampling frequencies, thus different Nyquist frequencies ( $f_{Nyq} = f_s/2$ ). The real signal components, smaller than the Nyquist frequency, will not depend on the sampling frequency, while the aliasing components will change/float with the changing Nyquist frequency (changing 'folding').



**Figure 4:** Measurement of a disk rotational speed by synchronizing stroboscope blinking light frequency so that reflective mark appears stationary (zero aliasing).

It is well-known, see Figure 4, that the strobe-synchronization will occur (a light-reflective mark on a rotating wheel appears stationary) if the wheel rotational speed (frequency),  $n$ , is the same as the blinking stroboscope (strobe) light frequency. However, it is less known, but still possible to measure a wheel's rotational speeds higher than the maximum strobe frequency, as explained next.



**Figure 5:** The "folding" diagram, representing the aliasing frequencies of the two signals with real frequencies higher than the Nyquist frequency

As described before, the strobe-synchronization will occur if the wheel rotational speed,  $n$ , is the same or an integer times bigger than the blinking stroboscope light frequency,  $n_j$ , i.e.  $n = j n_j$ , or  $n_j = n/j$ ; where  $j=1,2,3,\dots$ . Note that  $1Hz=60\text{ cpm}=60\text{ rpm}$ , where  $1\text{ rpm}$  (or  $1\text{ cpm}$ ) = 1 rotation (or cycle) per minute)

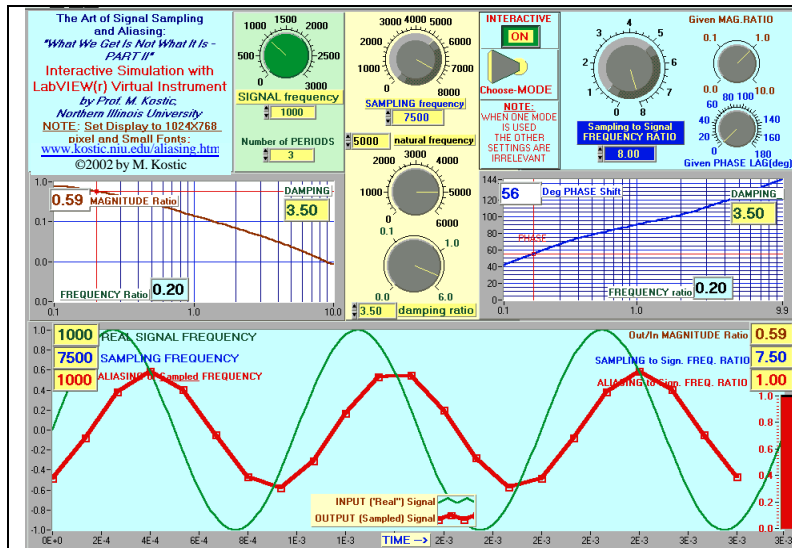
If we obtain any synchronization (synch) at  $n_1=n_1$  and consecutive smaller frequency synchronization at  $n_2, n_3, \dots n_i \dots n_N$ , then we may derive the real wheel speed,  $n$ , from any two strobe synchronization speeds (frequencies)<sup>[3]</sup>:

$$\text{Eq.(4)} \quad n = \frac{n_1 n_N (N - 1)}{n_1 - n_N}$$

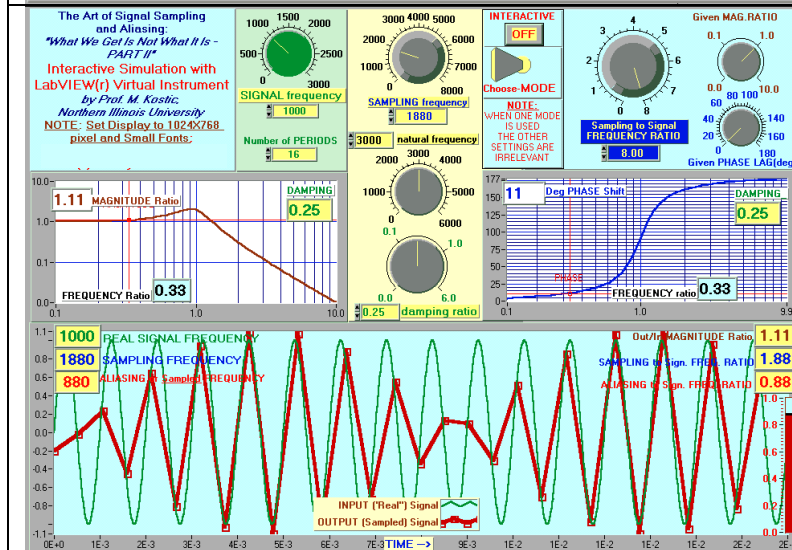
NOTE,  $N$  may be any number, so for  $N = 2$ :

$$\text{Eq.(5)} \quad n = \frac{n_1 n_2}{n_1 - n_2}$$

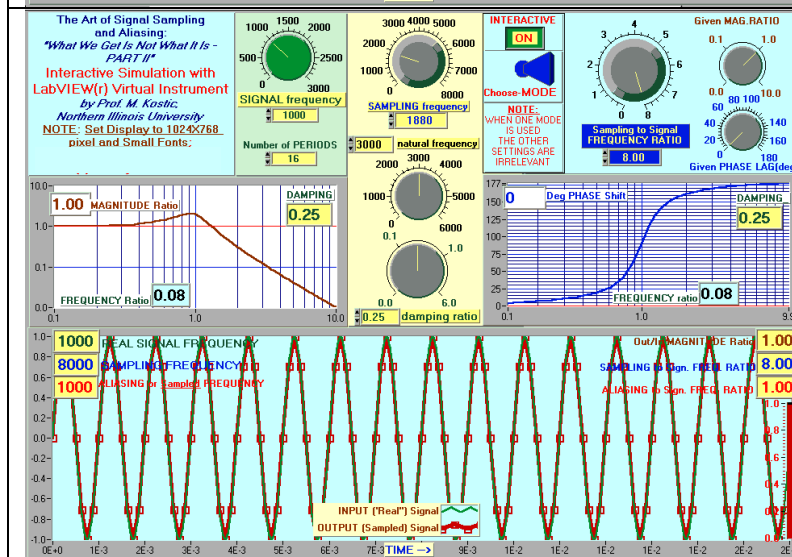
For example, for two measured, consecutive strobe synchronization-speeds,  $n_1=3000\text{ rpm}$  and  $n_2=2000\text{ rpm}$ , the real rotational speed will be,  $n = n_1 n_2 / (n_1 - n_2) = 3000 \times 2000 / (3000 - 2000) = 6000\text{ rpm}$ .



**Figure 6a:** Front Panel Interface:  
 “Choose-MODE”: Left-flip  
 “Input”:  
 Signal frequency: 1000 Hz  
 Number of periods: 3  
 Sampling frequency: 7500 Hz  
 Natural frequency: 5000 Hz  
 Damping ratio: 3.5  
 “Measured Output”:  
 Frequency ratio: 0.20  
 Magnitude ratio: 0.59  
 Phase lag (shift): 56 deg  
 Measured (aliasing) frequency:  
 1000 Hz  
 Sampling-to-signal frequency  
 ratio: 7.50  
 Aliasing-to-signal frequency  
 ratio: 1.00



**Figure 6b:** Front Panel Interface:  
 “Choose-MODE”: Left-flip  
 “Input”:  
 Signal frequency: 1000 Hz  
 Number of periods: 16  
 Sampling frequency: 1880 Hz  
 Natural frequency: 3000 Hz  
 Damping ratio: 0.25  
 “Measured Output”:  
 Frequency ratio: 0.33  
 Magnitude ratio: 1.11  
 Phase lag (shift): 11 deg  
 Measured (aliasing) frequency:  
 880 Hz  
 Sampling-to-signal frequency  
 ratio: 1.88  
 (close to 2, “Beat-wave”)  
 Aliasing-to-signal frequency  
 ratio: 0.88



**Figure 6c:** Front Panel Interface:  
 “Choose-MODE”: Right-flip  
 “Input”:  
 Signal frequency: 1000 Hz  
 Number of periods: 16  
 Sampling frequency: 8000 Hz  
 Natural frequency: 3000 Hz  
 Damping ratio: 0.25  
 “Measured Output”:  
 Frequency ratio: 0.08  
 Magnitude ratio: 1.00  
 Phase lag (shift): 0 deg  
 Measured (aliasing) frequency:  
 1000 Hz  
 Sampling frequency: 8000 Hz  
 Aliasing-to-signal frequency  
 ratio: 1.00

## Results and Discussion

As seen in Figure 6(a, b & c), the input variables are presented as the virtual instrument controls on its control panel. Several characteristic examples are presented on Figures 6a, 6b, and 6c, and are discussed next. As seen on these figures, the same signal (green thin curve) "measured" with different damping ratios and natural and sampling frequencies, appears quite differently in form, shape and even frequency (red thicker curve). This paper's theme phrase, "*What we 'see' is not what it is!*" is compellingly self-evident, especially during interactive presentation, or in a limited form, available as an interactive online experiment on the Web<sup>[4]</sup>. The simulations presented on Figure 6 expands on similar previous work<sup>[1]</sup> (also Figure 3), by accounting for the influence of instrument natural frequency and damping on the magnitude ratio and phase shift of the measured signal, in addition to influence of the sampling frequency on aliasing<sup>[5]</sup>.

It is evident in Figures 3 and 6 that the output signal does not represent the accurate image of the input signal, which is always the case to a smaller or a larger degree. For a signal of 1000 Hz, in Figure 6a for example, the sampling frequency of 7500 Hz (7.5 ratio) is good enough to avoid aliasing and get a good shape of the signal. However, the instrument natural frequency of 5000 Hz and 3.5 damping ratio are not good enough, thus attenuating the signal magnitude to 59% of its original value (0.59 magnitude ratio) and resulting in a phase lag of 56 degrees. In Figure 6b the undersampling with 1880 Hz, results in aliasing with measured (or aliasing) frequency of 880 Hz, and with a peculiar, so called "beat-wave" shape of frequency of 120 Hz, since the sampling-to-signal frequency ratio of 1.88 is smaller than 2 (the critical Nyquist ratio, thus aliasing), but close to 2 (thus "beat-wave" phenomena). The aliasing phenomena are explained elsewhere<sup>[1,4&5]</sup>, as well as related online experimentation, and is made available over the Internet<sup>[4]</sup>. Also, the small damping and signal-to-natural frequency ratios (0.25 and 0.33 respectively) result in a small resonance ( $1.11 > 1$  magnitude ratio) and a phase lag of 11 degrees. Finally, in Figure 6c, a good representation of the input signal is achieved by selecting the sampling-to-signal frequency ratio of 8 (no aliasing, good shape), magnitude ratio of 1 and phase shift or lag of 0, this time choosing the "right-flip" Choose-MODE feature, on the instrument front panel.

## Conclusion

The virtual instrument simulates the second-order (measurement) system to visualize and quantify data sampling and aliasing phenomena, an essential concept in many science and engineering courses. The stroboscope-mechanical system further visualizes the concept and shows a striking similarity with electronic data gathering and demonstrate aliasing. While students learn procedures to calculate the system frequency response (magnitude ratio and phase shift) as a function of damping and frequency ratios, many do not fully comprehend how these influence/modify a real-time signal output. As the aliasing phenomena are added to the measurement challenge, the students become confused and discouraged, and fail to get involved in order to fully understand these important physical processes. The virtual instrument enables students to experiment (or play) with different signals and instrument characteristics, and interactively see the relevant outcomes, the system/instrument frequency response and aliasing if present, along with input ("real") and output ("measured or sampled") signals, both qualitatively (visually/graphically) and quantitatively (numerically).

In addition to the development of an effective simulator and use of a stroboscope-mechanical system, a number of specific examples to illustrate sampling and aliasing were presented as typical and peculiar cases. Since, at our College of Engineering, a long-term effort is underway to promote *concurrent engineering* and develop interdisciplinary courses at all levels<sup>[6]</sup>, it is hoped that this learning simulator may be used in many engineering courses.

The philosophy of this work could be summed up into the following two focuses:

- Find-out: *facilitate inquiry and creativity by simulation, experimentation and observation.*
- Simplify: *be aware of complexity, but make it simple.*

The developed interactive simulator should stimulate users' curiosity and motivate them to conveniently "check out" different options and possibilities, and thus accelerate experience and by active, what-if inquiry and experimentations enhance their comprehension and interest for deeper understanding. These new learning tools are not and cannot replace the traditional learning by cognitive thinking, but when designed well and creatively used, the new simulation tools may qualitatively enhance the learning environment by stimulating inquiry and building confidence and motivation – in short, may become a "virtual eye and mind opener"!

Learning is a complex and challenging activity. New technology provides outstanding new opportunities for enhancing teaching and learning, but it also requires new and creative teaching methods, in fact a new paradigm. If new technology is used inappropriately (on the expense of proven traditional methods), it may be promoting itself only, thus wasting time and hurting rather than helping the learning process.

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