LINEAR CURVE FITTING

by **M. Kostic** 9/93,6/94

First, read the data from external files: DATA_xy.PRN, or DATAxy.PRN; or DATAx.PRN and DATAy.PRN, i.e.:

xy := READPRN("Data_xy")

NOTE: In MathCAD Ver.7 it should be: xy := READPRN("drive:\path\DATA_xy.prn")

If data are in two colomn-matrix, then, separate the **x** (0th) and the **y** (1st) colomns in separate vectors **x** and **y** (Remember that a matrix subscript starts from 0 in MathCAD):

$x := xy^{\langle 0 \rangle}$ $y := xy^{\langle 1 \rangle}$

or define data vectors **x** and **y**:

$$x := \begin{pmatrix} 1.4 \\ 4.7 \\ 17.3 \\ 82.9 \\ 171.6 \\ 1227.1 \end{pmatrix} \qquad y := \begin{pmatrix} 0.5 \\ 1.1 \\ 2.0 \\ 2.9 \\ 5.1 \\ 10.0 \end{pmatrix}$$

Evaluate the number of data points, or the size-length of vector **x** or **y**:

$N := \text{length}(x)$ $N = 6$

Then, data range variable will be:

$i := 0 .. N - 1$

Check (see) couple of data

$x_0 = 1.4$ $x_{N-1} = 1.227 \times 10^3$

$y_0 = 0.5$ $y_{N-1} = 10$

Compute sample statistics using the buit-in MathCAD functions:

$\text{mean}(x) = 250.833$ $\text{mean}(y) = 3.6$

$\text{var}(x) = 1.942 \times 10^5$ $\text{var}(y) = 10.353$

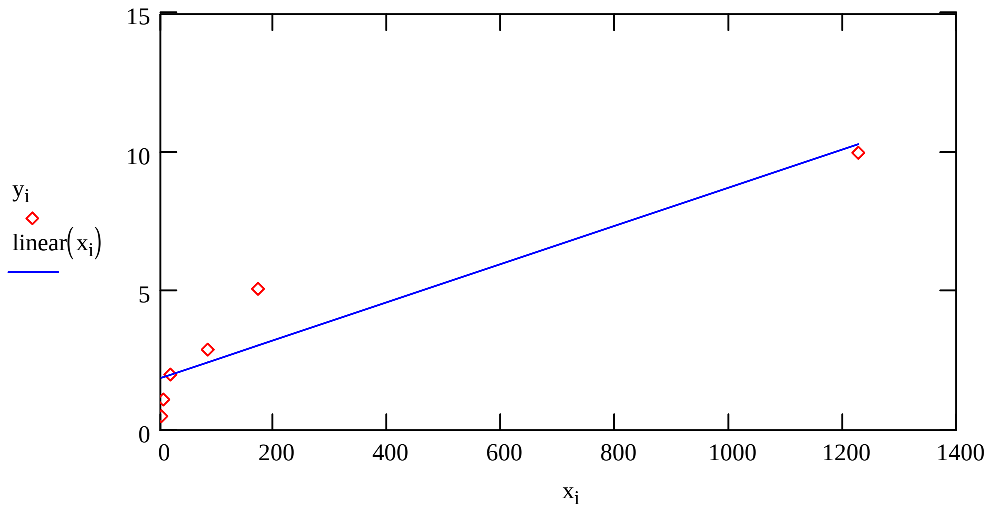
$\text{stdev}(x) = 440.627$ $\text{stdev}(y) = 3.218$

$\text{corr}(x, y) = 0.941$

$bl := \text{intercept}(x, y)$ $bl = 1.875$

$ml := \text{slope}(x, y)$ $ml = 6.875 \times 10^{-3}$ $\text{linear}(x) := bl + ml \cdot x$

Finally, plot the data and the fitted line:



15
10
5
0
0
200
400
600
800
1000
1200
1400
x_i
y_i
$linear(x_i)$

CURVE FITTING with POLYNOMIAL of m-ORDER

by **M. Kostic** 9/93, 6/95

NOTE: The nomenclature corresponds to *"Theory and Design for Mechanical Measurements, 2nd Ed."* by Figliola and Beasley, p.149, unless given otherwise.

For a polynomial of **m**-order (m=0,1,2,...N-1, may be **arbitrary** integer within this range):

$m := 2$ ...for example

Then, the range variable and the degree of freedom will be:

$j := 0..m \qquad \nu_m := m + 1 \qquad \nu := N - \nu_m$

For a polynomial curve fit:

$$y_c = a_0 + a_1 \cdot x + a_2 \cdot x^2 + .. + a_m \cdot x^m = \sum_{i=0}^{m} \left(a_i \cdot x^i\right)$$

and given data set {(x_i, y_i), **i=0,1,..N-1**} of **N** data points, we may determine coeficients a_i which will provide the best fit for the given conditions, using the least-square method.

Curve fit using matrix operations

Create **X** matrix ([N X (m+1)] order) first: $X^{\langle j \rangle} := \overrightarrow{x^j}$

and calculate (effectively) the polynomial coefficients:

$$a := \left(X^T \cdot X\right)^{-1} \cdot \left(X^T \cdot y\right)$$

...HOW SIMPLE!!

$$a = \begin{pmatrix} 0.945 \\ 0.027 \\ -1.584 \times 10^{-5} \end{pmatrix}$$

...the polynomial coefficients a_i

For "zero-order" polynomial we need a little "fix-up"

$$a_0 := \text{if}\left(m = 0, a, a_0\right)$$

Fitted curve (j=0...m) is:

$$y_{c_i} := \sum_j \left[a_j \cdot \left(x_i\right)^j\right]$$

Compute the *standard error of the fit* ***Sxy***:

$d := y - y_c \qquad D := \sum \overrightarrow{d^2} \qquad D_0 := \sum \overrightarrow{(y - \text{mean}(y))^2}$

$$Sxy := \sqrt{\frac{D}{N - \nu_m}} \qquad Sx0 := \sqrt{\frac{D_0}{N - 1}}$$

$Sxy = 0.453$

$Sx0 = 3.525$ is the Reference standard error for the "zero-order" fit, **Sy** in our Text

Correlation coefficient:

$$r := \sqrt{1 - \left(\frac{Sxy}{Sx0}\right)^2} \qquad r = 0.992$$

Now plot the data and the fitted curve to visualize the results : by **M. Kostic** 9/93, 6/95

If the number of data is smaller than, say Nc=60, plot the polynomial curve with at least **Nc** data

$Nc := 60$ $N = 6$ ...number of data points

$Nc := if(N < Nc, Nc, N)$ $dc := \frac{x_{N-1} - x_0}{Nc - 1}$ $Nc = 60$ ...number of points to plot the curve

$ic := 0 .. Nc - 1$ $m = 2$ ...order of polynomial

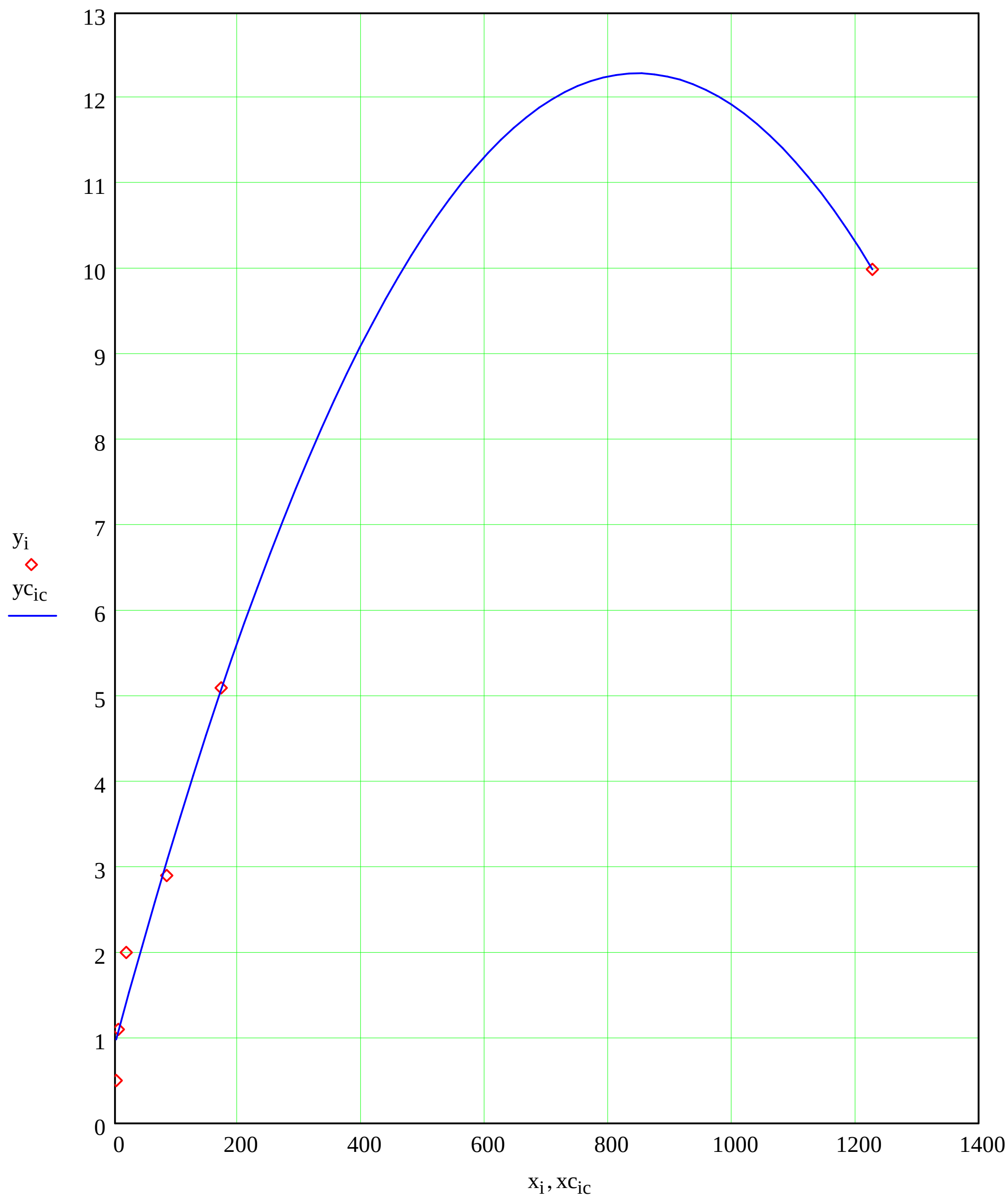
$xc_{ic} := x_0 + dc \cdot ic$

Check that these two are the same:

$$yc_{ic} := \sum_j \left[a_j \cdot \left(xc_{ic}\right)^j \right]$$

$x_{N-1} = 1.227 \times 10^3$

$xc_{Nc-1} = 1.227 \times 10^3$



DATAxy≡"Data-PRN\DataxyPoli.prn"