MEE 390/490 Problem 5.30, p.187/3rd Ed.

A temperature measurement system (termocouple for example) is calibrated against a standard system (mercury thermometer for example) certified to an uncertainty of +-0.05C at 95%. The system sensor is immersed alongside the standard within a temperature bath so that the two are separated by about 10 mm. The temperature uniformity of the bath is estimated at about 5C/m. The temperature system sensor is connected to a readout (multimeter for example) that indicates the temperature in terms of voltage. The following are calibration results between the temperature indicated by the standard **T[C]=y** and the indicated voltage **E[mV]=x**, i.e.:

$$x := \begin{pmatrix} 0.004 \\ 0.399 \\ 0.771 \\ 1.624 \\ 2.147 \\ 4.121 \end{pmatrix} \qquad y := \begin{pmatrix} 0.1 \\ 10.2 \\ 19.5 \\ 40.5 \\ 51.2 \\ 99.6 \end{pmatrix}$$

NOTE:
x = E[mV] =measured sensor's output electromotive force (voltage)
y = T[C] =measured temperature of the temperature standard

a) Compute the calibration curve fit. b) Estimate the uncertainty in using the output from the temperature measurement system for temperature measurements

Solution:

Evaluate the number of data points, or the size-length of vector **x** or **y**:

$N := \text{length}(x) \qquad N = 6$

Check (see) couple of data

$x_0 = 4\times 10^{-3} \qquad x_{N-1} = 4.121$

$y_0 = 0.1 \qquad y_{N-1} = 99.6$

Then, data range variable will be:

$i := 0..\, N-1$

Compute sample statistics using the buit-in MathCAD functions:

$\text{mean}(x) = 1.511 \qquad \text{mean}(y) = 36.85$

$\text{var}(x) = 1.881 \qquad \text{var}(y) = 1.086\times 10^{3}$

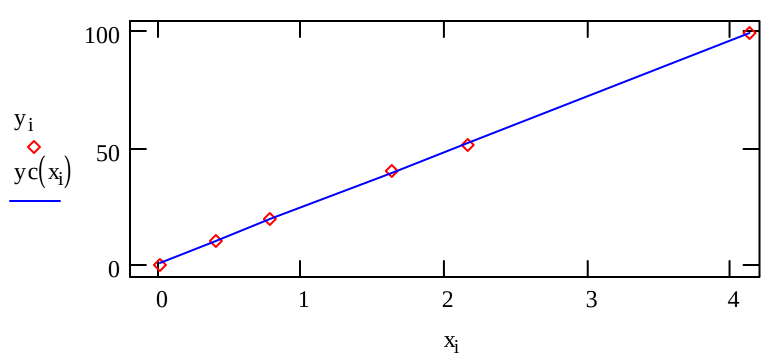
$\text{stdev}(x) = 1.371 \qquad \text{stdev}(y) = 32.961$

$\text{corr}(x,y) = 0.99983$

$b := \text{intercept}(x,y) \qquad b = 0.54$

$K := \text{slope}(x,y) \qquad K = 24.03$

Finally, plot the data and the fitted line: $yc(x) := b + K\cdot x$



LEGEND:
y =measured data
yc =calculated values using fitted curve

CURVE FITTING with POLYNOMIAL of m-ORDER

by **M. Kostic** 1993-1997

NOTE: The nomencluture corresponds to *"Theory and Design for Mechanical Measurements, 2nd Ed."* by Figliola and Beasley, p.149, unless given otherwise.

For a polynomial of **m**-order (m=0,1,2,...N-1, may be **arbitrary** integer within this range):

$m := 1$ ...for example

Then, the range variable and the degree of freedom will be:

$j := 0..m$ $\nu_m := m + 1$ $\nu := N - \nu_m$

For a polynomial curve fit:

$$y_c = a_0 + a_1 \cdot x + a_2 \cdot x^2 + .. + a_m \cdot x^m = \sum_{i=0}^{m} a_i \cdot x^i$$

and given data set **{(x_i,y_i), i=0,1,..N-1}** of **N** data points,

we may determine coeficients $\mathbf{a_i}$ which will provide the best fit for the given conditions, using the least-square method.

Curve fit using matrix operations

Create **X** matrix ([N X (m+1)] order) first: $X^{\langle j \rangle} := \overrightarrow{\left(x^j\right)}$

and calculate (effectively) the polynomial coefficients:

$$a := \left(X^T \cdot X\right)^{-1} \cdot \left(X^T \cdot y\right)$$ $$a = \begin{pmatrix} 0.54 \\ 24.03 \end{pmatrix}$$ ...the polynomial coefficients $\mathbf{a_i}$ $b = 0.54$ $K = 24.03$

...HOW SIMPLE!!

NOTE: $a_0 = b$ $a_1 = K$

For "zero-order" polynomial we need a little "fix-up"

$$a_0 := \text{if}\left(m = 0, a, a_0\right)$$

Fitted curve (j=0...m) is:

$$y_{c_i} := \sum_j a_j \cdot \left(x_i\right)^j$$

Compute the *standard error of the fit* ***Sxy***:

$$d := y - y_c \qquad D := \sum \overrightarrow{\left(d^2\right)} \qquad D_0 := \sum \overrightarrow{\left[(y - \text{mean}(y))^2\right]}$$

$$Sxy := \sqrt{\frac{D}{N - \nu_m}} \qquad Sx0 := \sqrt{\frac{D_0}{N - 1}}$$

$Sxy = 0.746$

$Sx0 = 36.107$ is the Reference standard error for the "zero-order" fit, **Sx0**=**Sy** in our Text

Correlation coefficient:

$$r := \sqrt{1 - \left(\frac{Sxy}{Sx0}\right)^2}$$ $r = 0.99979$

Calculation of uncertainties:

$B_{14} := 0.05$ ...due to uncertainty of the temperature standard

$B_{15} := 5 \cdot 0.01$ $\quad B_{15} = 0.05$ ...due to non-uniformity of the bath temerature (5C/m) for 10mm=0.01m separation distance

$K_{TE} := a_1$ $\quad K_{TE} = 24.03$ ...sensitivity (dT/dE) of temperature with regard to sensor's electromotive force

$B_{24} := 0.001 \cdot K_{TE}$ $\quad B_{24} = 0.024$ ...due to readout voltmetar uncertainty 0.001mV*24.03C/mv

$P_{31} := Sxy$ $\quad P_{31} = 0.746$ ...due to curve fit standard error

$\nu := 6 - \nu_m$ $\quad \nu = 4$ $\quad t_{4\_95} := 2.77$ ...Student-t value for 95% probability (From Table 4.4).

$$u_T := \sqrt{\left[\left(B_{14}^2 + B_{15}^2 + B_{24}^2\right) + \left(t_{4\_95} \cdot P_{31}\right)^2\right]}$$

$\sqrt{\left(B_{14}^2 + B_{15}^2 + B_{24}^2\right)} = 0.075$ ...bias part

$t_{4\_95} \cdot P_{31} = 2.066$ ...precision part

$u_T = 2.067$ ...total temperature uncertainty @ 95% probability

$a = \begin{pmatrix} 0.54 \\ 24.03 \end{pmatrix}$ **Final result**: $T(E) = 0.54 + 24.03 \cdot E$ with $u_T = 2.067$ with 95% probability, see plot below

Now plot the data and the fitted curve to visualize the results :

If the number of data is smaller than, say Nc=60, plot the polynomial curve with at least **Nc** data

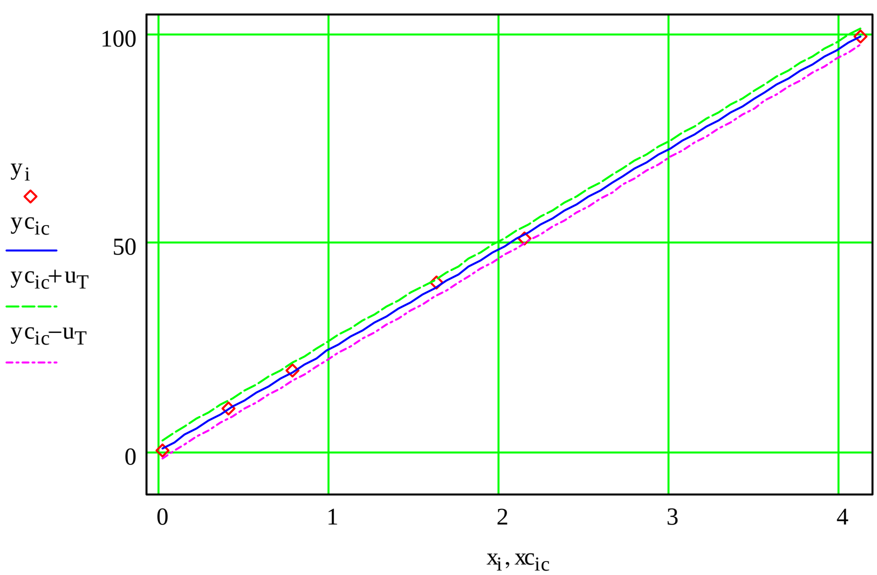
$Nc := 60$ $\quad N = 6$ ...number of data points

$Nc := if(N < Nc, Nc, N)$ $\quad dc := \dfrac{x_{N-1} - x_0}{Nc - 1}$ $\quad Nc = 60$ ...number of points to plot the curve

$ic := 0 .. Nc - 1$ $\quad m = 1$ ...order of polynomial

$xc_{ic} := x_0 + dc \cdot ic$ Check that these two are the same: $x_{N-1} = 4.121$ $\quad xc_{Nc-1} = 4.121$

$$yc_{ic} := \sum_j a_j \cdot \left(xc_{ic}\right)^j$$



LEGEND:
y =measured data
yc =calculated values using fitted curve
$yc+u_T$=upper limit
$yc - u_T$=lower limit