

From:

FLUID MECHANICS

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1–10 ■ ACCURACY, PRECISION, AND SIGNIFICANT DIGITS

In engineering calculations, the supplied information is not known to more than a certain number of significant digits, usually three digits. Consequently, the results obtained cannot possibly be precise to more significant digits. Reporting results in more significant digits implies greater precision than exists, and it should be avoided.

Regardless of the system of units employed, engineers must be aware of three principles that govern the proper use of numbers: accuracy, precision,

and significant digits. For engineering measurements, they are defined as follows:

- **Accuracy error** (*inaccuracy*) is the value of one reading minus the true value. In general, accuracy of a set of measurements refers to the closeness of the average reading to the true value. Accuracy is generally associated with repeatable, fixed errors.
- **Precision error** is the value of one reading minus the average of readings. In general, precision of a set of measurements refers to the fineness of the resolution and the repeatability of the instrument. Precision is generally associated with unrepeatable, random errors.
- **Significant digits** are digits that are relevant and meaningful.

A measurement or calculation can be very precise without being very accurate, and vice versa. For example, suppose the true value of wind speed is 25.00 m/s. Two anemometers A and B take five wind speed readings each:

Anemometer A: 25.50, 25.69, 25.52, 25.58, and 25.61 m/s. Average of all readings = 25.58 m/s.

Anemometer B: 26.3, 24.5, 23.9, 26.8, and 23.6 m/s. Average of all readings = 25.02 m/s.

Clearly, anemometer A is more precise, since none of the readings differs by more than 0.11 m/s from the average. However, the average is 25.58 m/s, 0.58 m/s greater than the true wind speed; this indicates significant **bias error**, also called **constant error** or **systematic error**. On the other hand, anemometer B is not very precise, since its readings swing wildly from the average; but its overall average is much closer to the true value. Hence, anemometer B is more accurate than anemometer A, at least for this set of readings, even though it is less precise. The difference between accuracy and precision can be illustrated effectively by analogy to shooting a gun at a target, as sketched in Fig. 1–40. Shooter A is very precise, but not very accurate, while shooter B has better overall accuracy, but less precision.

Many engineers do not pay proper attention to the number of significant digits in their calculations. The least significant numeral in a number implies the precision of the measurement or calculation. For example, a result written as 1.23 (three significant digits) *implies* that the result is precise to within one digit in the second decimal place; i.e., the number is somewhere between 1.22 and 1.24. Expressing this number with any more digits would be misleading. The number of significant digits is most easily evaluated when the number is written in exponential notation; the number of significant digits can then simply be counted, including zeroes. Some examples are shown in Table 1–3.

When performing calculations or manipulations of several parameters, the final result is generally only as precise as the least precise parameter in the problem. For example, suppose *A* and *B* are multiplied to obtain *C*. If *A* = 2.3601 (five significant digits), and *B* = 0.34 (two significant digits), then *C* = 0.80 (only two digits are significant in the final result). Note that most students are tempted to write *C* = 0.802434, with six significant digits, since that is what is displayed on a calculator after multiplying these two numbers.

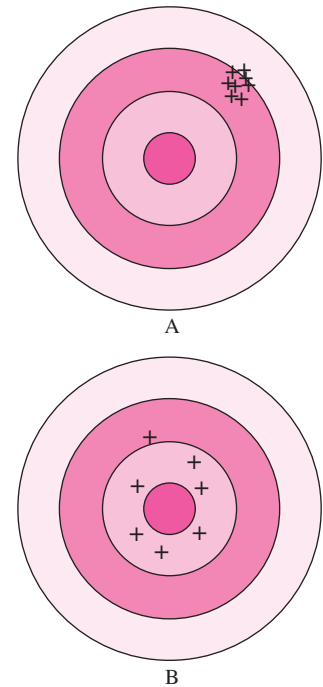


FIGURE 1–40

Illustration of accuracy versus precision. Shooter A is more precise, but less accurate, while shooter B is more accurate, but less precise.

TABLE 1–3

Significant digits

Number	Exponential Notation	Number of Significant Digits
12.3	1.23×10^1	3
123,000	1.23×10^5	3
0.00123	1.23×10^{-3}	3
40,300	4.03×10^4	3
40,300.	4.0300×10^4	5
0.005600	5.600×10^{-3}	4
0.0056	5.6×10^{-3}	2
0.006	$6. \times 10^{-3}$	1

Let's analyze this simple example carefully. Suppose the exact value of B is 0.33501, which is read by the instrument as 0.34. Also suppose A is exactly 2.3601, as measured by a more accurate and precise instrument. In this case, $C = A \times B = 0.79066$ to five significant digits. Note that our first answer, $C = 0.80$ is off by one digit in the second decimal place. Likewise, if B is 0.34499, and is read by the instrument as 0.34, the product of A and B would be 0.81421 to five significant digits. Our original answer of 0.80 is again off by one digit in the second decimal place. The main point here is that 0.80 (to two significant digits) is the best one can expect from this multiplication since, to begin with, one of the values had only two significant digits. Another way of looking at this is to say that beyond the first two digits in the answer, the rest of the digits are meaningless or not significant. For example, if one reports what the calculator displays, 2.3601 times 0.34 equals 0.802434, the last four digits are *meaningless*. As shown, the final result may lie between 0.79 and 0.81—any digits beyond the two significant digits are not only meaningless, but *misleading*, since they imply to the reader more precision than is really there.

As another example, consider a 3.75-L container filled with gasoline whose density is 0.845 kg/L, and determine its mass. Probably the first thought that comes to your mind is to multiply the volume and density to obtain 3.16875 kg for the mass, which falsely implies that the mass so determined is precise to six significant digits. In reality, however, the mass cannot be more precise than three significant digits since both the volume and the density are precise to three significant digits only. Therefore, the result should be rounded to three significant digits, and the mass should be reported to be 3.17 kg instead of what the calculator displays (Fig. 1-41). The result 3.16875 kg would be correct only if the volume and density were given to be 3.75000 L and 0.845000 kg/L, respectively. The value 3.75 L implies that we are fairly confident that the volume is precise within ± 0.01 L, and it cannot be 3.74 or 3.76 L. However, the volume can be 3.746, 3.750, 3.753, etc., since they all round to 3.75 L.

You should also be aware that sometimes we knowingly introduce small errors in order to avoid the trouble of searching for more accurate data. For example, when dealing with liquid water, we often use the value of 1000 kg/m³ for density, which is the density value of pure water at 0°C. Using this value at 75°C will result in an error of 2.5 percent since the density at this temperature is 975 kg/m³. The minerals and impurities in the water will introduce additional error. This being the case, you should have no reservation in rounding the final results to a reasonable number of significant digits. Besides, having a few percent uncertainty in the results of engineering analysis is usually the norm, not the exception.

When writing intermediate results in a computation, it is advisable to keep several "extra" digits to avoid round-off errors; however, the final result should be written with the number of significant digits taken into consideration. The reader must also keep in mind that a certain number of significant digits of precision in the result does not necessarily imply the same number of digits of overall *accuracy*. Bias error in one of the readings may, for example, significantly reduce the overall accuracy of the result, perhaps even rendering the last significant digit meaningless, and reducing the overall number of reliable digits by one. Experimentally determined values are

<input type="radio"/>	Given: Volume: $V = 3.75$ L
<input type="radio"/>	Density: $\rho = 0.845$ kg/L
	(3 significant digits)
	Also, $3.75 \times 0.845 = 3.16875$
	Find: Mass: $m = \rho V = 3.16875$ kg
<input type="radio"/>	Rounding to 3 significant digits:
	$m = 3.17$ kg
<input type="radio"/>	
<input type="radio"/>	

FIGURE 1-41

A result with more significant digits than that of given data falsely implies more precision.

subject to measurement errors, and such errors are reflected in the results obtained. For example, if the density of a substance has an uncertainty of 2 percent, then the mass determined using this density value will also have an uncertainty of 2 percent.

Finally, when the number of significant digits is unknown, the accepted engineering standard is three significant digits. Therefore, if the length of a pipe is given to be 40 m, we will assume it to be 40.0 m in order to justify using three significant digits in the final results.

EXAMPLE 1-6 Significant Digits and Volume Flow Rate

Jennifer is conducting an experiment that uses cooling water from a garden hose. In order to calculate the volume flow rate of water through the hose, she times how long it takes to fill a container (Fig. 1-42). The volume of water collected is $V = 1.1$ gal in time period $\Delta t = 45.62$ s, as measured with a stopwatch. Calculate the volume flow rate of water through the hose in units of cubic meters per minute.

SOLUTION Volume flow rate is to be determined from measurements of volume and time period.

Assumptions **1** Jennifer recorded her measurements properly, such that the volume measurement is precise to two significant digits while the time period is precise to four significant digits. **2** No water is lost due to splashing out of the container.

Analysis Volume flow rate \dot{V} is volume displaced per unit time and is expressed as

$$\text{Volume flow rate:} \quad \dot{V} = \frac{\Delta V}{\Delta t}$$

Substituting the measured values, the volume flow rate is determined to be

$$\dot{V} = \frac{1.1 \text{ gal}}{45.62 \text{ s}} \left(\frac{3.785 \times 10^{-3} \text{ m}^3}{1 \text{ gal}} \right) \left(\frac{60 \text{ s}}{1 \text{ min}} \right) = 5.5 \times 10^{-3} \text{ m}^3/\text{min}$$

Discussion The final result is listed to two significant digits since we cannot be confident of any more precision than that. If this were an intermediate step in subsequent calculations, a few extra digits would be carried along to avoid accumulated round-off error. In such a case, the volume flow rate would be written as $\dot{V} = 5.4759 \times 10^{-3} \text{ m}^3/\text{min}$. Based on the given information, we cannot say anything about the *accuracy* of our result, since we have no information about systematic errors in either the volume measurement or the time measurement.

Also keep in mind that good precision does not guarantee good accuracy. For example, if the batteries in the stopwatch were weak, its accuracy could be quite poor, yet the readout would still be displayed to four significant digits of precision.

In common practice, precision is often associated with *resolution*, which is a measure of how finely the instrument can report the measurement. For example, a digital voltmeter with five digits on its display is said to be more

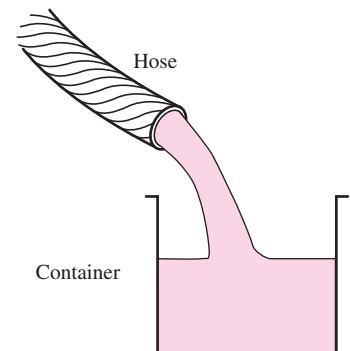
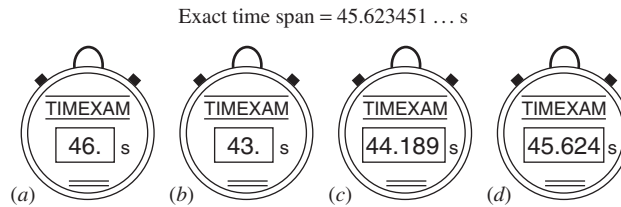


FIGURE 1-42

Schematic for Example 1-6 for the measurement of volume flow rate.

FIGURE 1–43

An instrument with many digits of resolution (stopwatch *c*) may be less accurate than an instrument with few digits of resolution (stopwatch *a*). What can you say about stopwatches *b* and *d*?



precise than a digital voltmeter with only three digits. However, the number of displayed digits has nothing to do with the overall *accuracy* of the measurement. An instrument can be very precise without being very accurate when there are significant bias errors. Likewise, an instrument with very few displayed digits can be more accurate than one with many digits (Fig. 1–43).

SUMMARY

In this chapter some basic concepts of fluid mechanics are introduced and discussed. A substance in the liquid or gas phase is referred to as a *fluid*. *Fluid mechanics* is the science that deals with the behavior of fluids at rest or in motion and the interaction of fluids with solids or other fluids at the boundaries.

The flow of an unbounded fluid over a surface is *external flow*, and the flow in a pipe or duct is *internal flow* if the fluid is completely bounded by solid surfaces. A fluid flow is classified as being *compressible* or *incompressible*, depending on the density variation of the fluid during flow. The densities of liquids are essentially constant, and thus the flow of liquids is typically incompressible. The term *steady* implies *no change with time*. The opposite of steady is *unsteady*, or *transient*. The term *uniform* implies *no change with location* over a specified region. A flow is said to be *one-dimensional* when the velocity changes in one dimension only. A fluid in direct contact with a solid surface sticks to the surface and

there is no slip. This is known as the *no-slip condition*, which leads to the formation of *boundary layers* along solid surfaces.

A system of fixed mass is called a *closed system*, and a system that involves mass transfer across its boundaries is called an *open system* or *control volume*. A large number of engineering problems involve mass flow in and out of a system and are therefore modeled as *control volumes*.

In engineering calculations, it is important to pay particular attention to the units of the quantities to avoid errors caused by inconsistent units, and to follow a systematic approach. It is also important to recognize that the information given is not known to more than a certain number of significant digits, and the results obtained cannot possibly be accurate to more significant digits. The information given on dimensions and units; problem-solving technique; and accuracy, precision, and significant digits will be used throughout the entire text.

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